6K p180

Problem Set #6

① If $r<1$ then a geometric series will be convergent

② S_4, S_7, S_∞ for each!

a) $144 + 48 + 16 + \cdots$

$r = \frac{1}{3}$

$S_4 = \frac{144\left(\left(\frac{1}{3}\right)^4 - 1\right)}{\frac{1}{3} - 1} = \boxed{\frac{640}{3}}$

$S_7 = \frac{144\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\frac{1}{3} - 1} = \boxed{\frac{17488}{81}}$

$S_\infty = \frac{144}{1 - \frac{1}{3}} = \boxed{216}$

b) $500 + 400 + 320 + \cdots$

$r = \frac{4}{5}$

$S_4 = \frac{500\left(\left(\frac{4}{5}\right)^4 - 1\right)}{\frac{4}{5} - 1} = \boxed{1476}$

$S_7 = \frac{500\left(\left(\frac{4}{5}\right)^7 - 1\right)}{\frac{4}{5} - 1} = \boxed{1975.712}$

$S_\infty = \frac{500}{1 - \frac{4}{5}} = \boxed{625}$

c) $80 + 8 + 0.8 + \cdots$

$r = \frac{1}{10}$

$S_4 = \frac{80\left(\left(\frac{1}{10}\right)^4 - 1\right)}{\frac{1}{10} - 1} = \boxed{\frac{2222}{25}}$

$S_7 = \frac{80\left(\left(\frac{1}{10}\right)^7 - 1\right)}{\frac{1}{10} - 1} = \boxed{88.88888}$

$S_\infty = \frac{80}{1 - \frac{1}{10}} = \boxed{\frac{800}{9}}$

d) $\frac{9}{2} + 3 + 2 + \cdots$

$r = \frac{2}{3}$

$S_4 = \frac{\frac{9}{2}\left(\left(\frac{2}{3}\right)^4 - 1\right)}{\frac{2}{3} - 1} = \boxed{\frac{65}{6}}$

$S_7 = \frac{\frac{9}{2}\left(\left(\frac{2}{3}\right)^7 - 1\right)}{\frac{2}{3} - 1} = \boxed{\frac{2059}{162}}$

$S_\infty = \frac{\frac{9}{2}}{1 - \frac{2}{3}} = \boxed{\frac{27}{2}}$

③

$S_\infty = \frac{27}{2}$

$S_3 = 13$

Find S_5

$S_\infty = \frac{27}{2}$

$\frac{u_1}{1-r} = \frac{27}{2}$

$u_1 = \frac{27}{2}(1-r)$

$S_3 = \frac{u_1(r^3-1)}{r-1}$

$13 = \frac{u_1(r^3-1)}{(r-1)}$

$13 = \frac{27}{2}(1-r)\cdot\frac{(r^3-1)}{(r-1)(-1)}$

$13 = -13.5(r^3-1)$

$13 = -13.5r^3 + 13.5$

$-0.5 = -13.5r^3$

$\frac{1}{27} = r^3$

$\frac{1}{3} = r$

$S_5 = \frac{u_1((\frac{1}{3})^5-1)}{\frac{1}{3}-1}$

$S_5 = \frac{\frac{27}{2}(1-\frac{1}{3})((\frac{1}{3})^5-1)}{\frac{1}{3}-1}$

$S_5 = \frac{121}{9}$

---

④

$u_3 = 24$

$u_6 = 3$

Find S_∞

$u_6 = u_3 r^3$

$3 = 24r^3$

$\frac{1}{8} = r^3$

$\frac{1}{2} = r$

$S_\infty = \frac{u_1}{1-r}$

$S_\infty = \frac{96}{1-\frac{1}{2}}$

$S_\infty = 192$

$u_3 = u_1 r^2$

$24 = u_1(\frac{1}{2})^2$

$(4)\,24 = u_1$

$96 = u_1$

⑤ $u_2 = 12$
$S_\infty = 64$
Find u_1

$S_\infty = \frac{u_1}{1-r}$ $\quad u_2 = u_1 r$

$S_\infty (1-r) = u_1$ $\quad \frac{u_2}{r} = u_1$

$64(1-r) = u_1$ $\quad \frac{12}{r} = u_1$

$64 - 64r = \frac{12}{r}$

$64r - 64r^2 = 12$

$0 = 64r^2 - 64r + 12$

$0 = 16r^2 - 16 + 3$

$0 = (4r-1)(4r-3)$

$r = \frac{1}{4}$ | $r = \frac{3}{4}$

2 solns.

If $r = \frac{1}{4}$,

$12 = u_1(\frac{1}{4})$

$\boxed{48 = u_1}$

If $r = \frac{3}{4}$

$12 = u_1(\frac{3}{4})$

$\boxed{16 = u_1}$

⑥ $r = 0.4$
$S_\infty = 250$
Find u_1

$S_\infty = \frac{u_1}{1-r}$

$250 = \frac{u_1}{1-0.4}$

$250 = \frac{u_1}{0.6}$

$\boxed{150 = u_1}$

⑦ $S_5 = 3798$
$S_\infty = 4374$
Find S_7

$4374 = \frac{u_1}{1-r}$ (S_∞)

$4374(1-r) = u_1$

$S_5 = \frac{u_1(r^5-1)}{r-1}$

Substitute

$3798 = \frac{4374(1-r)(r^5-1)}{(r-1)(-1)}$

$-3798 = 4374(r^5-1)$

$-3798 = 4374r^5 - 4374$

$576 = 4374r^5$

$\frac{576}{4374} = r^5$

$\frac{32}{243} = r^5$

$\frac{2}{3} = r$

$4374(1-\frac{2}{3}) = u_1$

$1458 = u_1$

$S_7 = \frac{1458((\frac{2}{3})^7 - 1)}{\frac{2}{3} - 1}$

$\boxed{S_7 = 4118}$