

## CHAPTER 3 SUMMARY

### Definitions

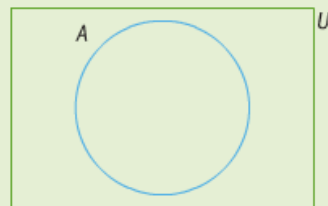
- An **event** is an outcome from an experiment.  
An **experiment** is the process by which we obtain an outcome.  
A **random experiment** is one where there is uncertainty over which event may occur.
- The theoretical probability of an event  $A$  is  $P(A) = \frac{n(A)}{n(U)}$   
where  $n(A)$  is the number of ways that event  $A$  can occur and  $n(U)$  is the total number of possible outcomes.
- If the probability of an event is  $P$ , in  $n$  trials you would expect the event to occur  $n \times P$  times.
- You can use relative frequency as an estimate of probability.  
The larger the number of trials, the closer the relative frequency is to the probability.

### Venn diagrams

- As an event,  $A$ , either happens or it does not happen

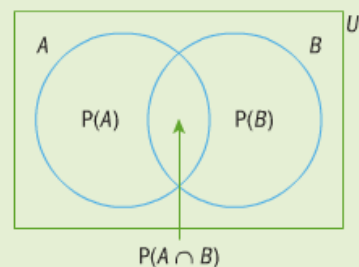
$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

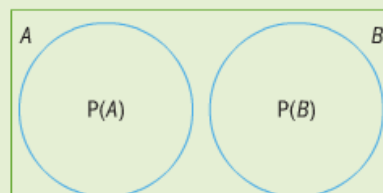


- For any two events  $A$  and  $B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



- In general, if  $A$  and  $B$  are mutually exclusive, then  $P(A \cap B) = 0$  and  $P(A \cup B) = P(A) + P(B)$ .





## Sample space diagrams and the product rule

- Two events  $A$  and  $B$  are independent if the occurrence of one does not affect the chance that the other occurs.
- When two events  $A$  and  $B$  are independent  
$$P(A \cap B) = P(A) \times P(B)$$

This is the **product rule for independent events**.  
This is also called the multiplication rule.

- Don't forget that a sample space diagram is usually a chart!!

## Conditional probability

- For two events  $A$  and  $B$  the probability of  $A$  occurring given that  $B$  has occurred can be found using

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- If  $A$  and  $B$  are independent events,  $P(A|B) = P(A)$ ,  $P(B|A) = P(B)$ ,  
 $P(A|B') = P(A)$ ,  $P(B|A') = P(B)$
- In general for two events  $A$  and  $B$  the probability of  $A$  occurring given that  $B$  has occurred can be found using

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



## Review exercise

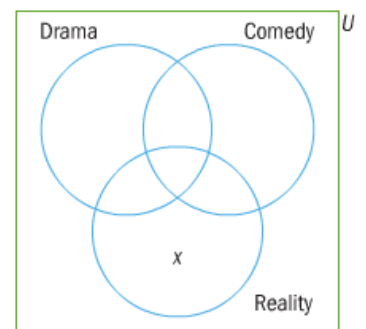
- 1 A two-digit number between 10 and 99 inclusive is written down at random. What is the probability that it:
  - a is divisible by 5,
  - b is divisible by 3,
  - c is greater than 50,
  - d is a square number?
- 2 In a class of 30 students, 18 have a dog, 20 have a cat and 3 have neither. A student is selected at random. What is the probability that this student has both a cat and a dog?

### EXAM-STYLE QUESTIONS

- 3 For events  $C$  and  $D$  it is known that:  
 $P(C) = 0.7$     $P(C' \cap D') = 0.25$     $P(D) = 0.2$ .
  - a Find  $P(C \cap D')$ .
  - b Explain why  $C$  and  $D$  are not independent events.
- 4 The two events  $A$  and  $B$  are such that  $P(A) = 0.6$ ,  $P(B) = 0.2$  and  $P(A|B) = 0.1$ .  
Calculate the probabilities that:
  - a both of the events occur,
  - b at least one of the events occur,
  - c exactly one of the events occur,
  - d  $B$  occurs given that  $A$  has occurred.
- 5 A group of 100 students are asked which of three types of TV programme, drama, comedy and reality, they watch regularly. They provide the following information:
  - 15 watch all three types;
  - 18 watch drama and comedy;
  - 22 watch comedy and reality TV;
  - 35 watch drama and reality TV;
  - 10 watch of none these programmes regularly.

There are three times as many students who watch drama only than comedy only and two times as many who watch comedy only than reality TV only.

- a If  $x$  is taken as the number of students who watch reality TV only, write an expression for the number of students who watch drama only.
- b Using all the above information copy and complete the Venn diagram.
- c Calculate the value of  $x$ .





## Review exercise

- 1 Let  $P(C) = 0.4$ ,  $P(D) = 0.5$ ,  $P(C | D) = 0.6$ .
  - a Find  $P(C \text{ and } D)$ .
  - b Are  $C$  and  $D$  mutually exclusive? Give a reason for your answer.
  - c Are  $C$  and  $D$  independent events? Give a reason for your answer.
  - d Find  $P(C \text{ or } D)$ .
  - e Find  $P(D | C)$ .
  
- 2 Jack does  $\frac{3}{5}$  of the jobs around the house and Jill does the rest. If 35% of Jack's jobs are finished properly and 55% of Jill's jobs are finished properly, find the probability that a job done around the house will be done:
  - a properly,
  - b by Jill if it was not done properly.

### EXAM-STYLE QUESTIONS

- 3 Max travels to school each day by bicycle, by bus or by car. The probability that he travels by bus on any day is 0.6. The probability that he travels by bicycle on any day is 0.3.
  - a Draw a tree diagram which shows the possible outcomes for Max's journeys on Monday and Tuesday. Label the tree clearly, writing in the probabilities of each outcome.
  - b What is the probability that he travels
    - i by bicycle on Monday and Tuesday,
    - ii by bicycle on Monday and by bus on Tuesday,
    - iii by the same method of travel on Monday and Tuesday?
  - c Max traveled to school by bicycle on Monday and Tuesday. What is the probability that he does not travel to school by bicycle on Wednesday and Thursday and Friday?
  - d What is the probability that in any three days Max travels twice by car and once by bus or twice by bicycle and once by car?
  
- 4 A bag contains 6 red apples and 10 green apples. Without looking into the bag, Maddy randomly selects one apple.
  - a What is the probability that it is red?

The apple is red and Maddy eats it. Next the bag is passed to Janet. Without looking into the bag, she randomly selects one apple.
  - b What is the probability that it is green?

The apple is green and Janet replaces it in the bag. Next the bag is passed to Tarish. Without looking into the bag, he randomly selects two apples.
  - c What is the probability that they are both red?