Exponents and Logarithms – REVIEW Name_____

REVIEW – Exponential and Logarithmic Functions

1. List the domain and range for the function $f(x) = \log_6 |x|$.

2. Matching – Put the letter of the asymptote next to the function it corresponds to. You may use some asymptotes twice and some asymptotes may not be used at all.

_____ $f(x) = -8\log_2 x$

_____ $f(x) = e^{(x-3)}$

_____ $f(x) = \log_3 x$

_____ $f(x) = -e^x - 1$

_____ $f(x) = \log(x+4)$

Asymptotes

a) $y = -4$

b) $x = -4$

c) $y = 0$

d) $x = 0$

e) $y = 3$

f) $x = 3$

g) $y = -1$

3. During a four year span, the value of a home depreciated by 4% in the first year, by 7% in the second year, by 1% in the third year, and by 2% in the fourth year. What is the overall percent of depreciation of the house over the entire four year period? Round you answer to the nearest hundredth of a percent.

4. Recall that carbon 14 has a half life of 5,700 years. If a living thing dies, what percentage of its carbon 14 will remain after 1000 years?

A fossil is found that contains 70% of its original carbon 14. Approximately how old is the fossil?

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5. Solve each equation.

a) $5^x = 4^{x+6}$

b) $\log(x+1)+\log(x-1)=\log 8$

6. Write the single logarithm as the sum or difference of multiple logarithms. No exponents are allowed.

$$\ln\sqrt{\frac{x^3}{yz}}$$

7. In the early stages of a measles epidemic there were 100 infected people and each day the number rose by 10%.
   - **a** How many people were infected
     - **i** after 2 days
     - **ii** after a week?
   - **b** How long would it take for 250 people to be infected?

8. Joseph did a parachute jump for charity. After jumping out of the aircraft his velocity at time t seconds after his parachute opened was v m s^{-1} where
   $v = 9 + 29e^{-0.063t}$
   - **a** Sketch the graph of v against t.
   - **b** What was Joseph's speed at the instant the parachute opened?
   - **c** What was his lowest possible speed if he fell from a very great height?
   - **d** If he actually landed after 45 seconds what was his speed on landing?
   - **e** How long did it take him to reach half the speed he had when the parachute opened?

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9. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

10. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

11. Solve the equation $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

Bonus – (Do on separate paper)

Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation.

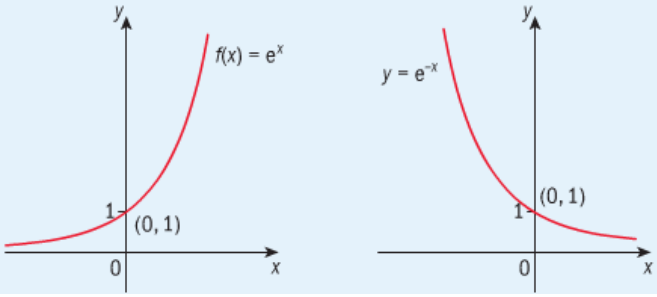
CHAPTER 4 SUMMARY
Exponents

Laws of exponents

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{(a^m)} = (\sqrt[n]{a})^m = \left(a^m\right)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}}$
- $a^{-n} = \dfrac{1}{a^n}$

Exponential functions

- An **exponential function** is a function of the form $f(x) = a^x$ where a is a positive real number (that is, $a > 0$) and $a \neq 1$.
- The **domain** of the exponential function is the set of all real numbers.
- The **range** is the set of all positive real numbers.
- The graph of the exponential function $f(x) = e^x$ is a graph of exponential growth and the graph of $f(x) = e^{-x}$ is a graph of exponential decay.



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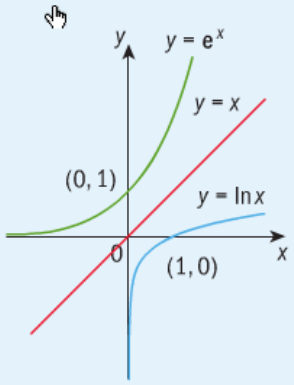
Logarithms

Properties of logarithms

- If $b = a^x$ then $\log_a b = x$
- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a b$ is undefined for any base a if b is negative
- $\log_a 0$ is undefined
- $\log_a (a^n) = n$

Logarithmic functions

- To find an **inverse** of a function algebraically, switch x and y and then rearrange to make y the subject.
- Generally if $f : x \mapsto a^x$ then $f^{-1} : x \mapsto \log_a x$
  $y = \log_a x$ is the inverse of $y = a^x$.
- $y = \ln x$ is the inverse of the exponential function $y = e^x$



- $\log_a (a^x) = x$ and $a^{\log_a x} = x$
  $\ln(e^x) = x$ and $e^{\ln x} = x$
  $\log(10^x) = x$ and $(10^{\log x}) = x$

Laws of logarithms

- $\log x + \log y = \log xy$
- $\log x - \log y = \log \frac{x}{y}$
- $\log x^n = n \log x$
- $\log \frac{1}{x} = -\log x$

Change of base formula

- $\log_b a = \frac{\log_c a}{\log_c b}$