Problem Set #6

15 B

① X = square of die face

Find $E(x)$

x	1	4	9	16	25	36
$P(X=x)$	1/6	1/6	1/6	1/6	1/6	1/6

$$E(x) = 1(\tfrac{1}{6}) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6)$$

$$= 91/6 \approx 15.2 \text{ (3sf)}$$

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② $E(z) = 2(1/6) + 3(1/6) + 5(1/6) + 7(x) + 11(y)$

$$5\tfrac{2}{3} = 10/6 + 7x + 11y$$

$$\frac{17}{3} = \frac{5}{3} + 7x + 11y$$

$$\frac{12}{3} = 7x + 11y$$

$$4 = 7x + 11y$$

Remember probabilities add up to 1:

* so also, $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + x + y = 1$

$$x + y = \frac{1}{2}$$

→ system of equations ←

$$7x + 11y = 4 \xrightarrow{\times 2} 14x + 22y = 8$$

$$2x + 2y = 1 \xrightarrow{\times -7} -14x - 14y = -7$$

$$8y = 1$$

$$\boxed{y = \tfrac{1}{8}}$$

$$x + y = \frac{1}{2}$$

$$x + \tfrac{1}{8} = \frac{1}{2}$$

$$\boxed{x = \frac{3}{8}}$$

③

x	1	2	3	5	8	13
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(x) = 1(1/6) + 2(1/6) + 3(1/6) + 5(1/6) + 8(1/6) + 13(1/6)$

$E(x) = 32/6 \approx 5.33$ (3sf)

④ $P(x) = \frac{x}{36}$

x	1	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$\frac{8}{36}$

$E(x) = 1(1/36) + 2(2/36) + 3(3/36) + 4(4/36) + 5(5/36) + 6(6/36)$
$+ 7(7/36) + 8(8/36)$

$E(x) = 204/36 \approx 5.67$ (3sf)

⑤

x	1	2	3	4	5	6	7	8	9
$P(X=x)$	1K	2K	3K	4K	5K	4K	3K	2K	1K

a) $25K = 1$

$K = \frac{1}{25}$

b) $E(x) = 1(1/25) + 2(2/25) + 3(3/25) + 4(4/25) + 5(5/25) + 6(4/25)$
$+ 7(3/25) + 8(2/25) + 9(1/25)$

$E(x) = 125/25 = 5$

⑥

x	1	2	3
$P(X=x)$	0.2	$1-k$	

a) k must be < 0.8 since $P(1) = 0.2$

$0.2 \leq k < 0.8$

$0.2 + 1 - k + c = 1$

$-0.2 \quad -1 \quad +k \qquad -1 \quad -0.2 \quad +k$

$c = k - 0.2$ ← * k must be ≥ 0.2 or else $P(3)$ would be negative

b) $E(x) = 1(0.2) + 2(1-k) + 3(k-0.2)$

$E(x) = 0.2 + 2 - 2k + 3k - 0.6$

$E(x) = k + 1.6$

$\boxed{E(x) - 1.6 = k}$