Bell  Ringer

Where do you think this function will have problems with its graph?

$$f(x) = \frac{2x+1}{x^2-x-2}$$

at $x=2$

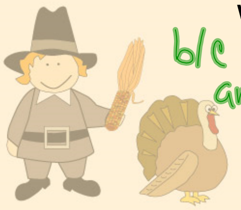
Why?

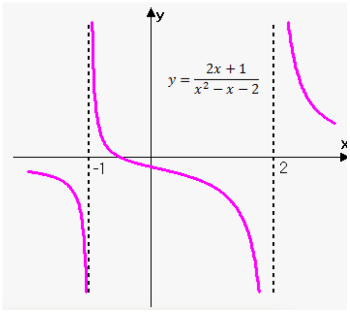
b/c it causes an undefined

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2 \quad x=-1$





Nov 20-11:15 AM

RATIONAL FUNCTIONS

A rational function is a function of the form:

$$R(x)=\frac{p(x)}{q(x)}$$

where *p* and *q* are polynomials

Nov 20-11:27 AM

What would the domain of a rational function be?

We'd need to make sure the denominator $\neq 0$ $$R(x) = \frac{p(x)}{\boxed{q(x)}}$$

Find the domain.

$$R(x) = \frac{5x^2}{\boxed{3+x}} \qquad \{x \in \Re : x \neq -3\}$$

$$H(x) = \frac{x-3}{\boxed{(x+2)(x-2)}} \qquad \{x \in \Re : x \neq -2, x \neq 2\}$$

$$F(x) = \frac{x-1}{\boxed{x^2+5x+4}}$$

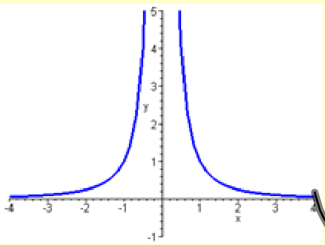
$$(x+4)(x+1) = 0$$

If you can't see it in your head, set the denominator = 0 and factor to find "illegal" values.

$$\{x \in \Re : x \neq -4, x \neq -1\}$$

Nov 20-11:27 AM

The graph of $f(x) = \frac{1}{x^2}$ looks like this:

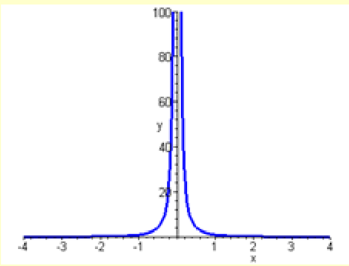


$\frac{1}{x}$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

V-asymptote



If you choose x values close to 0, the graph gets close to the asymptote, but never touches it.

Since $x \neq 0$, the graph approaches 0 but never crosses or touches 0. A vertical line drawn at $x = 0$ is called a **vertical asymptote**. It is a sketching aid to figure out the graph of a rational function. There will be a vertical asymptote at x values that make the denominator = 0

Nov 20-11:27 AM

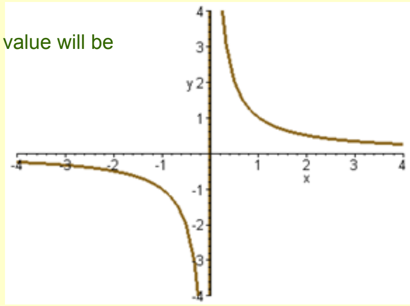
Let's consider the graph $f(x)=\frac{1}{x}$

We recognize this function as the reciprocal function from our "library" of functions.

Can you see the vertical asymptote?

Let's see why the graph looks like it does near 0 by putting in some numbers close to 0.

The closer to 0 you get for *x* (from positive direction), the larger the function value will be



$$f\left(\frac{1}{10}\right)=\frac{1}{\frac{1}{10}}=10 \qquad f\left(\frac{1}{100}\right)=\frac{1}{\frac{1}{100}}=100$$

Try some negatives

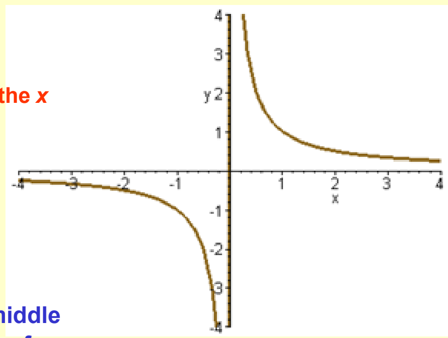
$$f\left(-\frac{1}{10}\right)=\frac{1}{-\frac{1}{10}}=-10 \qquad f\left(-\frac{1}{100}\right)=\frac{1}{-\frac{1}{100}}=-100$$

Nov 20-11:27 AM

Does the function $f(x)=\frac{1}{x}$ have an *x* intercept? $0 \neq \frac{1}{x}$

There is NOT a value that you can plug in for *x* that would make the function = 0. The graph approaches but never crosses the horizontal line *y* = 0. This is called a horizontal asymptote.

A graph will NEVER cross a vertical asymptote because the *x* value is "illegal" (would make the denominator 0)



A graph may cross a horizontal asymptote near the middle of the graph but will approach it when you move to the far right or left

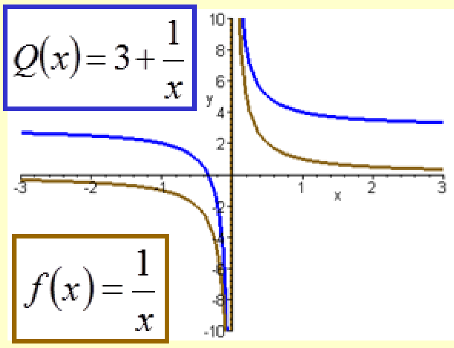
Nov 20-11:27 AM

Graph $Q(x) = 3 + \frac{1}{x} = \frac{1}{x} + 3$

vertical translation,

moved up 3

This is just the reciprocal function transformed. We can trade the terms places to make it easier to see this.



The vertical asymptote remains the same because in either function, x ≠ 0

The horizontal asymptote will move up 3 like the graph does.

Nov 20-11:27 AM

Finding Asymptotes

There will be a vertical asymptote at any "illegal" *x* value, so anywhere that would make the denominator = 0 * unless that also makes the numerator = 0

$$R(x) = \frac{x^2 + 2x + 5}{x^2 - 3x - 4}$$

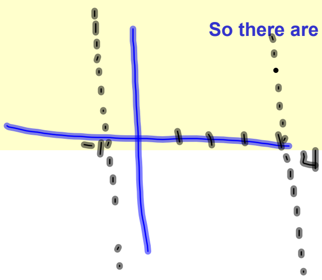
Let's set the bottom = 0 and factor and solve to find where the vertical asymptote(s) should be.

$$(x-4)(x+1) = 0$$

$x = 4$ $x = -1$

VERTICAL ASYMPTOTES

So there are vertical asymptotes at *x* = 4 and *x* = -1.



Roots are when numerator = 0

Nov 20-11:27 AM

HORIZONTAL ASYMPTOTES

We compare the degrees of the polynomial in the numerator and the polynomial in the denominator to tell us about horizontal asymptotes.

degree of top = 1

$$R(x) = \frac{2x^1 + 5}{x^2 - 3x + 4}$$

degree of bottom = 2

If the degree of the numerator is less than the degree of the denominator, (remember degree is the highest power on any *x* term) the *x* axis is a horizontal asymptote.

$$1 < 2$$

Nov 20-11:27 AM

HORIZONTAL ASYMPTOTES

The leading coefficient is the number in front of the highest powered *x* term.

degree of top = 2

$$R(x) = \frac{2x^2 + 4x + 5}{1x^2 - 3x + 4}$$

degree of bottom = 2

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at:

$y = \dfrac{\text{leading coefficient of top}}{\text{leading coefficient of bottom}}$

horizontal asymptote at:

$$y = \frac{2}{1} = 2$$

Nov 20-11:27 AM

SUMMARY OF HOW TO FIND ASYMPTOTES

Vertical Asymptotes are the values that are NOT in the domain. To find them, set the denominator = 0 and solve.

To determine horizontal or oblique asymptotes, compare the degrees of the numerator and denominator.

- **If the degree of the top < the bottom, horizontal asymptote along the *x* axis (*y* = 0)**
- **If the degree of the top = bottom, horizontal asymptote at *y* = leading coefficient of top over leading coefficient of bottom**
- **If the degree of the top > the bottom, oblique asymptote found by long division.**

Nov 20-11:27 AM