

1. Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

(Total 4 marks)

2. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

(Total 6 marks)

3. A curve has equation $y = x(x - 4)^2$.

(a) For this curve find

- (i) the x -intercepts;
- (ii) the coordinates of the maximum point;
- (iii) the coordinates of the point of inflexion.

(9)

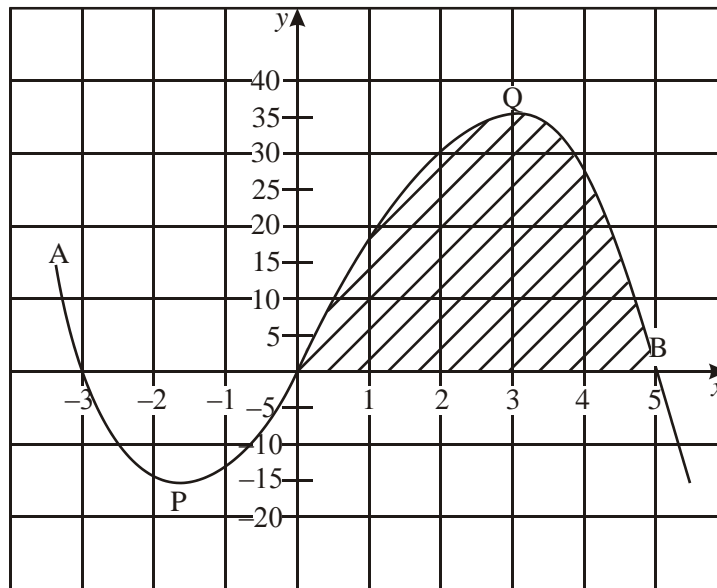
(b) Use your answers to part (a) to sketch a graph of the curve for $0 \leq x \leq 4$, clearly indicating the features you have found in part (a).

(3)

(Total 12 marks)

4. The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x -axis at $A(-3,0)$, $B(5,0)$ and the origin, O . There is a minimum point at P and a maximum point at Q .

(a) The function may also be written in the form $f : x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a ;

(ii) b .

(2)

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q .

(7)

(c) (i) Find the equation of the tangent to the graph of f at O .

(ii) This tangent cuts the graph of f at another point. Give the x -coordinate of this point.

(4)

(Total 13 marks)

5. A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2, t \geq 0$$

- (a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released). (2)
- (b) Show that the height of the ball after one second is 17 metres. (2)
- (c) At a later time the ball is **again** at a height of 17 metres.
- (i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.
- (ii) Solve the equation **algebraically**. (4)
- (d) (i) Find $\frac{dh}{dt}$.
- (ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).
- (iii) Find **when** the ball reaches its maximum height.
- (iv) Find the maximum height of the ball.

(7)
(Total 15 marks)

6. Differentiate with respect to x :

(a) $\ln(3x - 1)$

(b) $\sqrt{3 - 4x}$

(c) $(2x + 5)^3$

(Total 6 marks)

7. A particle moves along a straight line. When it is a distance s from a fixed point, where $s > 1$, the velocity v is given by $v = \frac{(3s + 2)}{(2s - 1)}$. Find the acceleration when $s = 2$.

(Total 4 marks)