

Derivation of Vertex Form:

lets start with

$$y = ax^2 + bx + c$$

$$\left(\frac{b}{2a}\right)^2$$

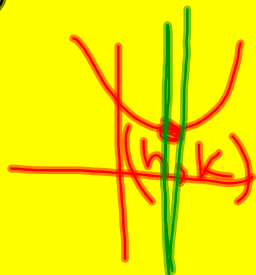
$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{ab^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{ab^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

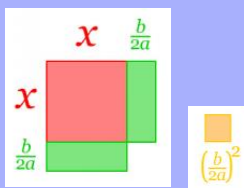
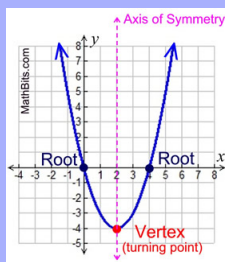
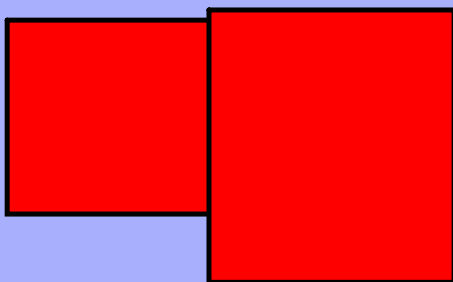
$$y = a(x-h)^2 + k$$



$$-h = \frac{b}{2a}$$

$$h = -\frac{b}{2a}$$

What are our tools for solving quadratics?

Derivation of Quadratic Formula:

lets start with

$$0 = ax^2 + bx + c$$

$$0 = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$\left(\frac{b}{2a}\right)^2$

$$0 = x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)$$

$$\frac{b^2}{4a^2} - \frac{4ac}{4a^2} = x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right)$$

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \sqrt{\left(x + \frac{b}{2a}\right)^2}$$

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$\frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x$$

$$\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = x$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$