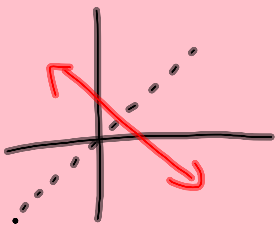
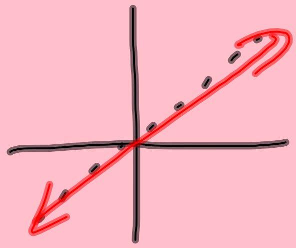
What is $f^{-1}(x)$ if

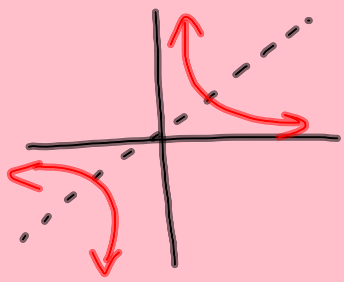
a $f(x) = 1 - x$



b $f(x) = x$



c $f(x) = \frac{1}{x}, x \neq 0$



What do all of these functions have in common graphically?

Sep 16-10:12 AM

① $f(x) = 3x - 1$

$y = 3x - 1$

inverse: $x = 3y - 1$

$x + 1 = 3y$

$\frac{x+1}{3} = y$

$f^{-1}(x) = \frac{x+1}{3}$

Sep 18-12:04 PM

1 b) $g(x) = x^3 - 2$

$y = x^3 - 2$

<u>inverse:</u> $x = y^3 - 2$

$x + 2 = y^3$

$\sqrt[3]{x+2} = y$

$g^{-1}(x) = \sqrt[3]{x+2}$

Sep 18-12:06 PM

1c) $h(x) = \frac{1}{4}x + 5$

$y = \frac{1}{4}x + 5$

<u>inverse:</u> $x = \frac{1}{4}y + 5$

$x - 5 = \frac{1}{4}y$

$4(x-5) = y$

or $h^{-1}(x) = 4(x-5)$

$h^{-1}(x) = 4x - 20$

Sep 18-12:08 PM

1d) $f(x) = \sqrt[3]{x} - 3$

$y = \sqrt[3]{x} - 3$

inverse: $x = \sqrt[3]{y} - 3$

$x + 3 = \sqrt[3]{y}$

$(x+3)^3 = y$

$f^{-1}(x) = (x+3)^3$

Sep 18-12:11 PM

1e) $g(x) = \frac{1}{x} - 2$

$y = \frac{1}{x} - 2$

inverse $x = \frac{1}{y} - 2$

$\frac{x+2}{1} = \frac{1}{y}$

$\frac{y\cancel{(x+2)}}{\cancel{x+2}} = \frac{1}{x+2}$

$y = \frac{1}{x+2}$

$g^{-1}(x) = \frac{1}{x+2}$

Sep 18-12:13 PM

1 f) $h(x) = 2x^3 + 3$

$$y = 2x^3 + 3$$

inverse: $x = 2y^3 + 3$

$$x - 3 = 2y^3$$

$$\frac{x-3}{2} = y^3$$

$$\sqrt[3]{\frac{x-3}{2}} = y$$

$$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

Sep 18-12:16 PM

1g) $f(x) = \frac{x}{3+x}$

$$y = \frac{x}{3+x}$$

inverse $\frac{x}{1} = \frac{y}{3+y}$

$$y = x(3+y)$$

$$y = 3x + xy$$

$$y - xy = 3x$$

$$y(1-x) = 3x$$

$$y = \frac{3x}{1-x}$$

$$f^{-1}(x) = \frac{3x}{1-x}$$

Sep 18-12:19 PM

1h) $g(x) = \frac{2x}{5-x}$

$y = \frac{2x}{5-x}$

inverse: $\frac{x}{1} = \frac{2y}{5-y}$

$2y = x(5-y)$

$2y = 5x - xy$

$2y + xy = 5x$

$y(2+x) = 5x$

$y = \frac{5x}{2+x}$

$g^{-1}(x) = \frac{5x}{2+x}$

Sep 18-1:01 PM

2) $f^{-1}(5)$

$f(x) = 6 - x$

$y = 6 - x$

inverse $x = 6 - y$

$x - 6 = -y$

$-x + 6 = y$

$f^{-1}(x) = -x + 6$

$f^{-1}(5) = -5 + 6$

$= \boxed{1}$

Sep 18-1:03 PM

2b) $f(x) = \frac{10}{x+7}$

$y = \frac{10}{x+7}$

inverse: $\frac{x}{1} = \frac{10}{y+7}$

$10 = x(y+7)$

$10 = xy + 7x$

$\frac{-7x+10}{x} = \frac{\cancel{x}y}{\cancel{x}}$

$f^{-1}(x) = \frac{-7x+10}{x}$

$f^{-1}(5) = \frac{-7(5)+10}{5}$

$= \frac{-35+10}{5}$

$= \frac{-25}{5}$

$= \boxed{-5}$

Sep 18-1:04 PM

3 $f(x) = \frac{x+1}{x-2}$

$y = \frac{x+1}{x-2}$

inverse $\frac{x}{1} = \frac{y+1}{y-2}$

$y+1 = x(y-2)$

$y+1 = xy - 2x$

$y - xy = -2x - 1$

$\frac{y(\cancel{1-x})}{(\cancel{1-x})} = \frac{-2x-1}{(1-x)}$

$f^{-1}(x) = \frac{-2x-1}{1-x} \begin{matrix}(-1)\\(-1)\end{matrix}$

or

$= \frac{2x+1}{x-1}$

Sep 18-1:06 PM

~~original~~

$y = mx + b$

Sep 18-1:08 PM

1.6 Transforming functions

What are transformations that you know from Geometry?

reflection

translation

rotation

dilation

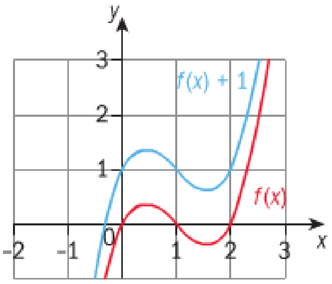
Sep 16-10:12 AM

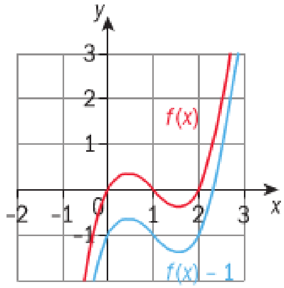
Translations

Shift upward or downward

→ $f(x) + k$ translates $f(x)$ vertically a distance of k units upward.

→ $f(x) - k$ translates $f(x)$ vertically a distance of k units downward.





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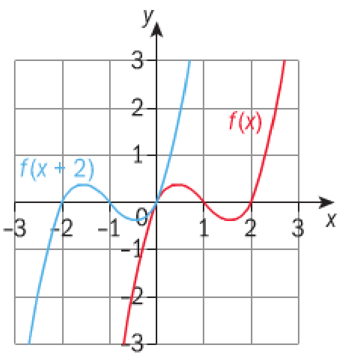
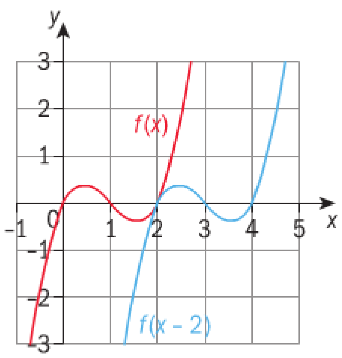
Shift to the right or left

→ $f(x + k)$ translates $f(x)$ horizontally k units to the **left**, when $k > 0$.

→ $f(x - k)$ translates $f(x)$ horizontally k units to the **right**, when $k > 0$.





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Translations can be indicated by a vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

This means right (or left) by a units and up (or down) by b units.

Ex) $f(x) = x^2$ is translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. What is the new equation and what is the effect on the graph?

$\leftarrow 3$

$\downarrow 2$

$g(x) = (x+3)^2 - 2$

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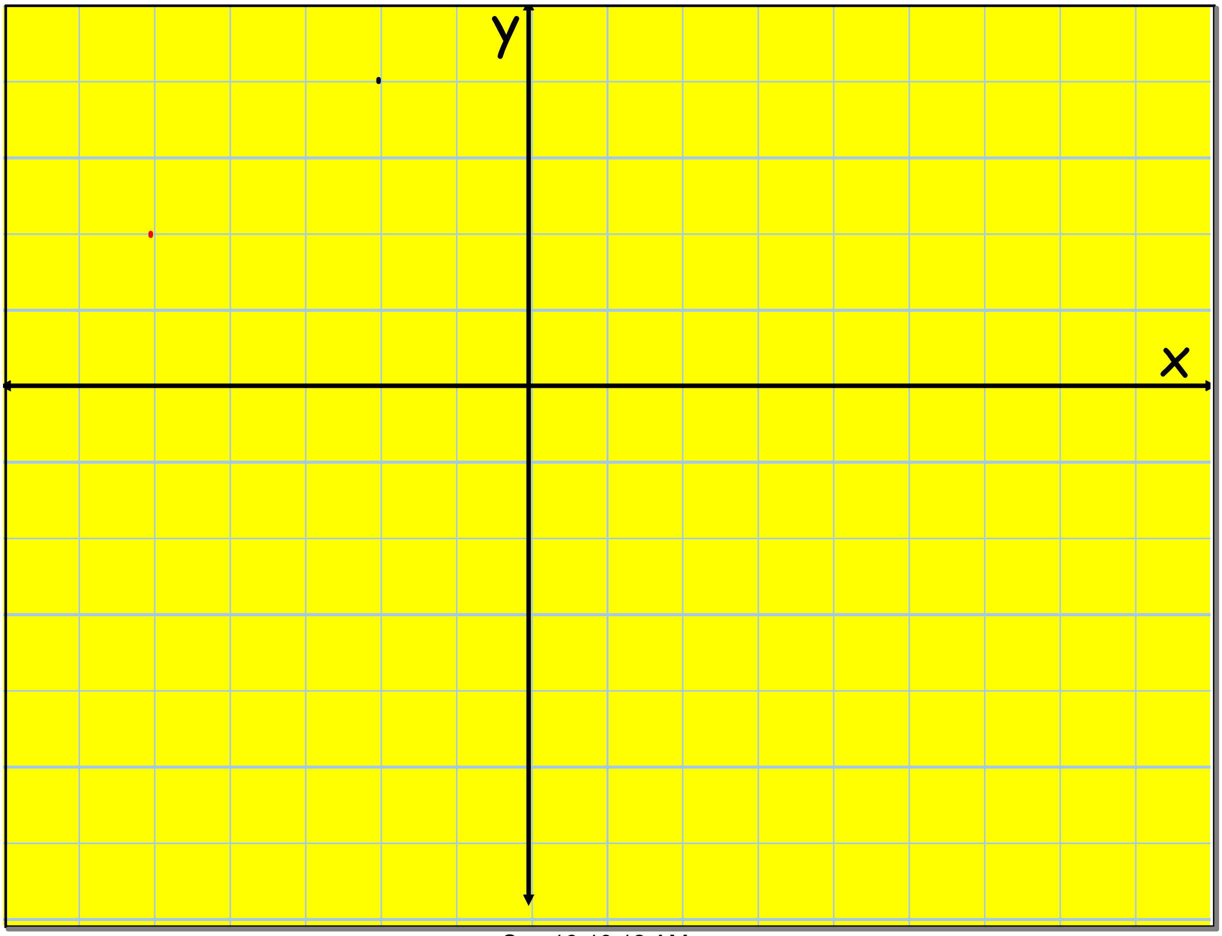

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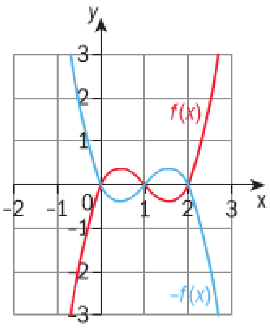
Reflections

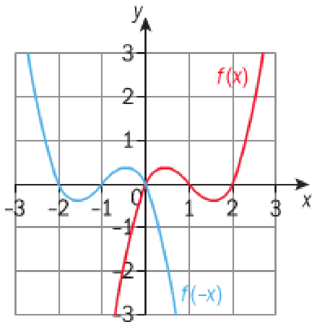
Reflection in the x-axis

→ $-f(x)$ reflects $f(x)$ in the x-axis.



Reflection in the y-axis

→ $f(-x)$ reflects $f(x)$ in the y-axis.



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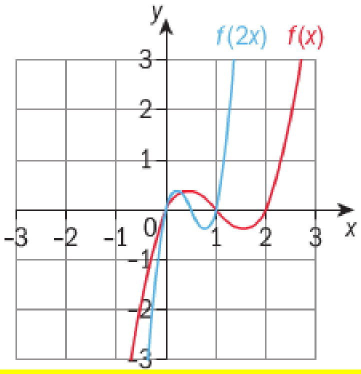
Stretches

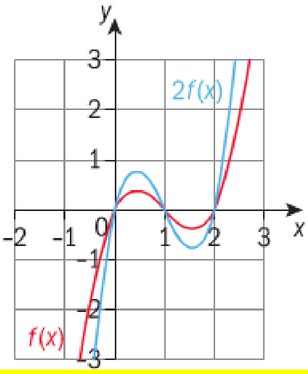
Horizontal stretch (or compress)

→ $f(qx)$ stretches or compresses $f(x)$ horizontally with scale factor $\frac{1}{q}$.



Vertical stretch (or compress)

→ $pf(x)$ stretches $f(x)$ vertically with scale factor p.



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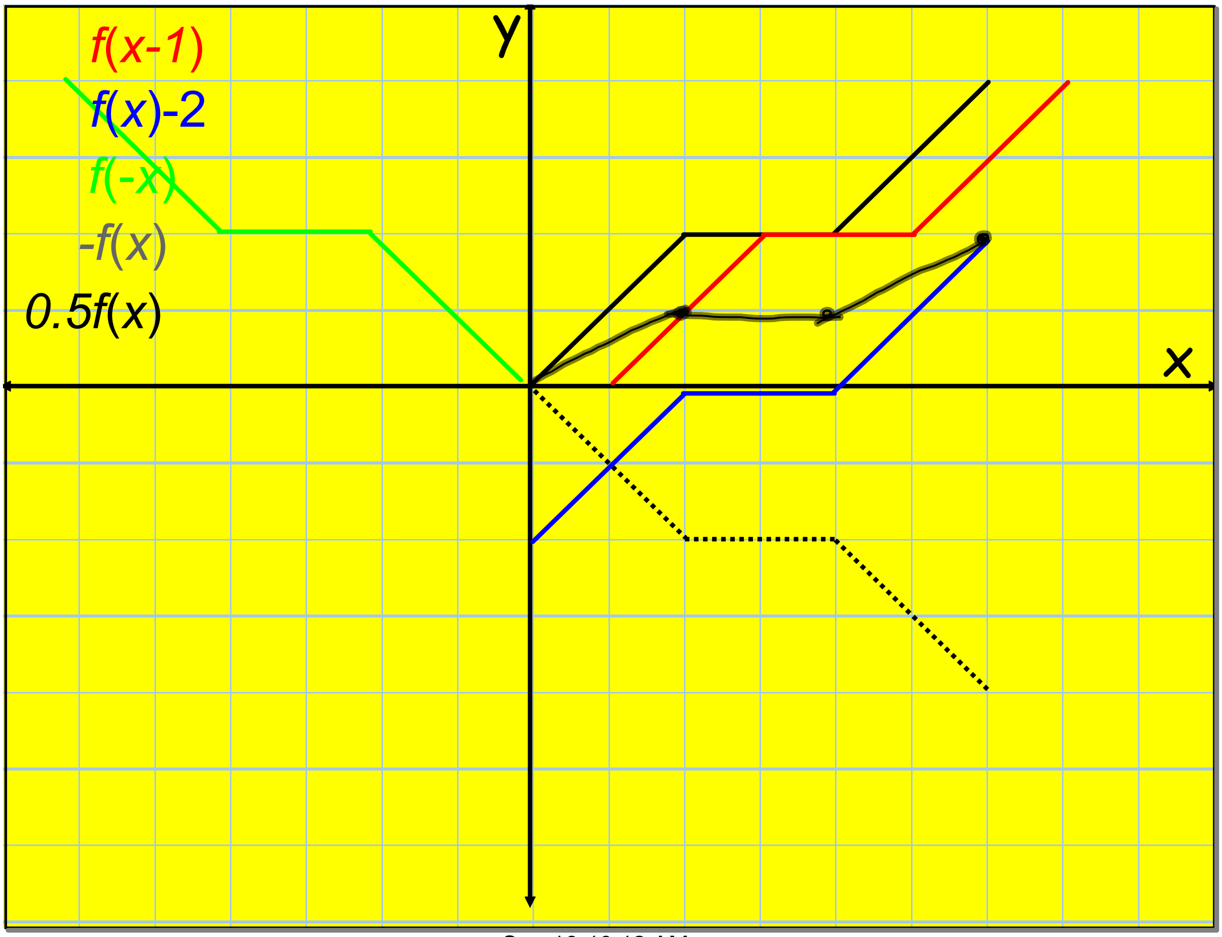

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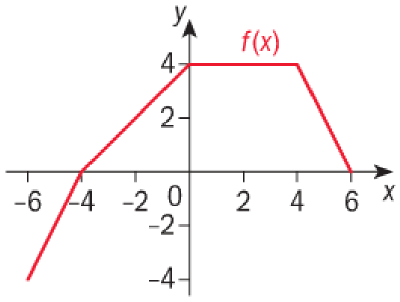
EXAM-STYLE QUESTION

1 Copy the graph. Draw these functions on the same axes.

a $f(x) + 4$ **b** $f(x) - 2$ **c** $-f(x)$

d $f(x + 3)$ **e** $f(x - 4)$ **f** $2f(x)$

g $f(2x)$



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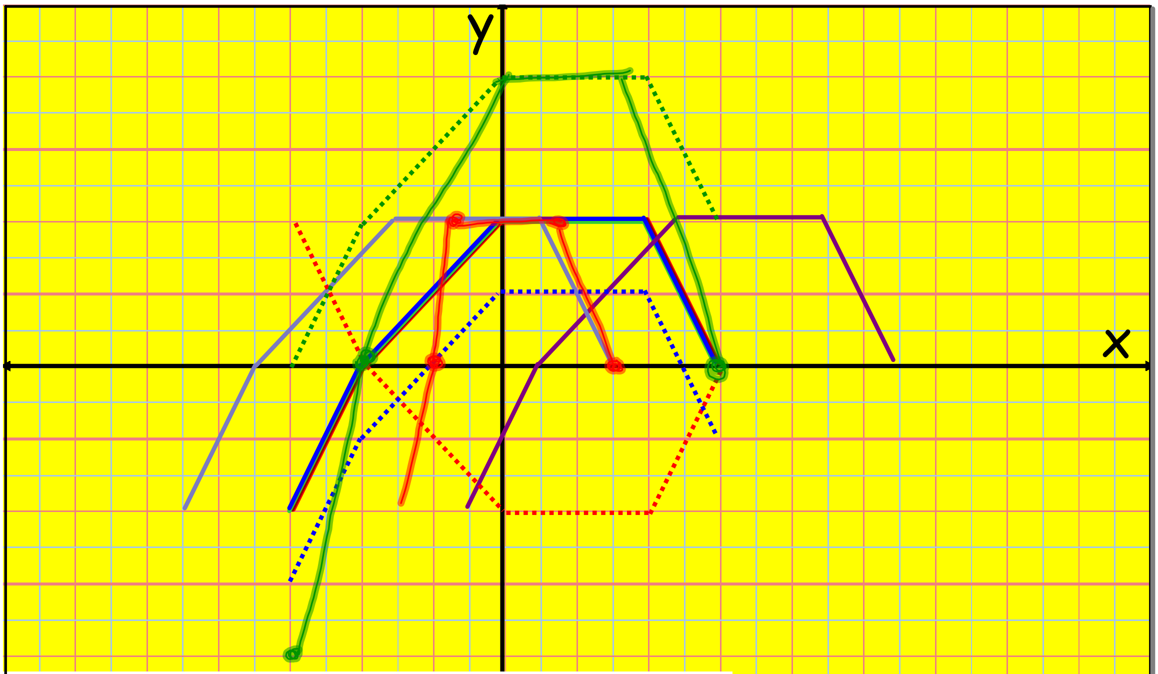

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