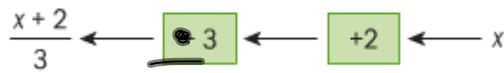


Finding inverse functions algebraically

Look at how the function $f(x) = 3x - 2$ is made up. We start with x on the left.



To form the inverse function we reverse the process, using inverse operations.



So $f^{-1}(x) = \frac{x+2}{3}$



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→ To find the inverse function algebraically, replace $f(x)$ with y and solve for y .

If $f(x) = 4 - 3x$, find $f^{-1}(x)$.

$$y = 4 - 3x$$

inverse : $x = 4 - 3y$

$$\frac{x-4}{-3} = \frac{-3y}{-3}$$

$$f^{-1}(x) = \frac{x-4}{-3} = -\left(\frac{x-4}{3}\right) = \frac{4-x}{3}$$

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→ The function $I(x) = x$ is called the identity function.
 It leaves x unchanged.
 So $f \circ f^{-1} = I$

$$f(x) = 4 - 3x$$

$$f^{-1}(x) = \frac{4-x}{3}$$

$$\begin{aligned} & f(f^{-1}(x)) \\ & f\left(\frac{4-x}{3}\right) \\ & 4 - 3\left(\frac{4-x}{3}\right) \\ & 4 - 4 + x \\ & x \quad \checkmark \end{aligned}$$

$$\begin{aligned} & f^{-1}(f(x)) \\ & f^{-1}(4-3x) \\ & \frac{4-(4-3x)}{3} \\ & \frac{4-4+3x}{3} \\ & x \quad \checkmark \end{aligned}$$

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EXAM-STYLE QUESTION

1 If $f(x) = \frac{x+4}{2}$ and $g(x) = 2x - 4$, find

a i $g(1)$ and $(f \circ g)(1)$

$$\begin{aligned} g(1) &= 2(1) - 4 \\ &= -2 \\ f(g(1)) &= f(-2) \\ &= \frac{-2+4}{2} = 1 \end{aligned}$$

ii $f(-3)$ and $(g \circ f)(-3)$

$$= (g \circ f)(-3) = -3$$

iii $(f \circ g)(x)$

$$(f \circ g)(x) = x$$

iv $(g \circ f)(x)$

$$(g \circ f)(x) = x$$

What does this tell us about functions f and g ?

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What is $f^{-1}(x)$ if

a $f(x) = 1 - x$

b $f(x) = x$

c $f(x) = \frac{1}{x}, x \neq 0$

What do all of these functions have in common graphically?

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