4.6 Laws of logarithms

We can deduce the laws of logarithms from the exponential equations, $x = a^p$ and $y = a^q$.

$x = a^p$ and $y = a^q$

then $p = \log_a x$ and $q = \log_a y$

and $xy = a^p \times a^q = a^{p+q}$

so $\log_a xy = p + q$

and hence $\log_a xy = \log_a x + \log_a y$

$xy = a^{p+q}$

$\log_a(xy) = \log_a(a^{p+q})$

→ $\log x + \log y = \log xy$

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$$\frac{x}{y} = a^p \div a^q = a^{p-q}$$

so $\log_a \frac{x}{y} = p - q$

and hence $\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a(a^{p-q})$

→ $\log x - \log y = \log \frac{x}{y}$

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$p = \log_a x$

$$x^n = (a^p)^n = a^{pn}$$

so $\log_a x^n = pn$

and hence $\log_a x^n = n \log_a x$

$\log_a(x^n) = \log_a(a^{pn})$

→ $n \log x = \log x^n$

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Express $\log_2 5 + \frac{1}{2}\log_2 36 - \log_2 10$ as a single logarithm.

Answer

$$\log_2 5 + \frac{1}{2}\log_2 36 - \log_2 10$$

$$= \log_2 5 + \log_2 36^{\frac{1}{2}} - \log_2 10$$

$$= \log_2 5 + \log_2 6 - \log_2 10$$

$$= \log_2 30 - \log_2 10 = \log_2\left(\frac{5 \cdot 6}{10}\right)$$

$$= \log_2 3$$

$n \log_a x = \log_a x^n$

$\log x + \log y = \log xy$

$\log x - \log y = \frac{x}{y}$

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