

Natural logarithms

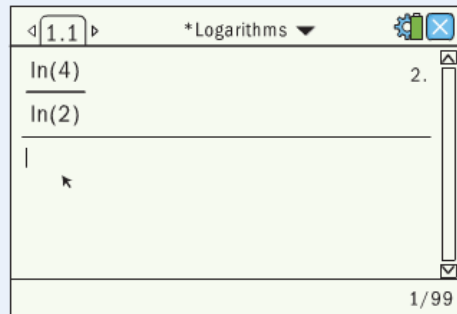
The **natural logarithm**, $\log_e x$ (log to the base e), is the other important logarithm.

You write $\ln x$ for $\log_e x$. There is an 'ln' key on the calculator

Use a calculator to evaluate $\frac{\ln 4}{\ln 2}$

Answer

$$\frac{\ln 4}{\ln 2} = 2$$



Oct 29-11:57 AM

$\rightarrow \log(a^x) = x$ and $a^{\log_a x} = x$

$\log_e(e^x) = x$ and $e^{\ln x} = x$

$\log(10^x) = x$ and $(10^{\log x}) = x$

$\rightarrow y = \ln x$ is the inverse of the exponential function $y = e^x$

The graph shows the exponential function $y = e^x$ (green curve) and the logarithmic function $y = \ln x$ (blue curve) as inverse functions. A red line $y = x$ represents the identity function. The points $(0, 1)$ and $(1, 0)$ are marked on the axes. Handwritten notes in green and blue identify the curves.

$\log_{\text{poop}}(\text{poop}^x) = x$

$(\text{poop})^{\log_{\text{poop}} x} = x$

Decorative footer with two pumpkins and a fence.

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Solve these equations, giving your answers to 3 significant figures.

a $e^x = 2.3$

b $\ln x = -1.5$

c $10^x = 0.75$

d $\log x = 3$

Answers

a $e^x = 2.3$

~~$\ln(e^x) = \ln 2.3$~~

$x = 0.833(3 \text{ sf})$

b $\ln x = -1.5$

~~$e^{\ln x} = e^{-1.5}$~~

$x = 0.223(3 \text{ sf})$

c $10^x = 0.75$

~~$\log(10^x) = \log 0.75$~~

$x = -0.125(3 \text{ sf})$

d $\log x = 3$

~~$10^{\log x} = 10^3$~~

$x = 1000$

$\log_e 2.3 = x$

$\ln 2.3 = x$

$e^{-1.5} = x$

Write in natural log form.

Use $\ln(e^x) = x$ and evaluate.

Use $(e^{\ln x}) = x$ and evaluate.

Use $\log(10^x) = x$ and evaluate.

Use $10^{\log x} = x$ and evaluate.

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Given that $f(x) = \frac{1}{3}e^{2x}$, find $f^{-1}(x)$.

Answer

$f(x) = \frac{1}{3}e^{2x}$

$y = \frac{1}{3}e^{2x}$

$x = \frac{1}{3}e^{2y}$

$3x = e^{2y}$

~~$\ln(3x) = \ln e^{2y}$~~

$\ln(3x) = 2y$

$\frac{1}{2}\ln(3x) = y$

So $f^{-1}(x) = \frac{1}{2}\ln(3x), x > 0$

Interchange x and y .

Use $\ln(e^x) = x$.

Solve for y .

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