

4.4 Properties of logarithms

$$\log_2 8$$

$$2^3 = 8$$

$$\log_2 8 = 3$$

$$\log_a 8 = 8$$

$$2^3 = 8$$

$$\rightarrow \text{If } b = a^x \text{ then } \log_a b = x$$

We read this as "log base a of b is x"
or in human terms :

"the exponent that a needs on it to be worth b is x."



Oct 26-8:37 AM

Evaluate $\log_5 125 = x$

Answer

Evaluate $\log_{64} 4$

Answer

$$x = \log_{64} 4$$

$$64^x = 4$$

$$(4^3)^x = 4^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

Change equation to exponent form.

Write 64 as 4^3

*Equate the exponents
and solve for x.*



Oct 26-10:50 AM

Evaluate $\log_4 4$

Answer

$$x = \log_4 4$$

$$4^x = 4$$

$$x = 1$$

*Write 'x =' log statement.
Change equation to exponent
form.
Equate exponents ($4 = 4^1$).*

In general, the log to base a of any number $a = 1$.

$$\rightarrow \log_a a = 1$$



Oct 26-10:59 AM

Evaluate $\log_5 1$

Answer

$$x = \log_5 1$$

$$5^x = 1$$

$$x = 0$$

Write equation in exponent form.

Any number raised to the power 0 is equal to 1 so ...

$$\rightarrow \log_a 1 = 0$$



Oct 26-11:02 AM

Some log expressions are **undefined** – this means that you can't find solutions for them.

- 1 What happens when you try to evaluate the expression

$$\log_3(-27)?$$

First write the log equation.

$$x = \log_3(-27)$$

Then rewrite the equation in exponent form.

$$3^x = -27$$

This equation has no solution.

→ $\log_a b$ is undefined for any base a if b is negative.



- 2 What is the value of $\log_3 0$?

First write an equation.

$$x = \log_3 0$$

Rewrite in exponent form.

$$3^x = 0$$

This equation has no solution.

→ $\log_a 0$ is undefined.

Oct 26-11:04 AM

Evaluate $\log_2 2^5$

Answer

$$x = \log_2 2^5$$

$$2^x = 2^5$$

$$x = 5$$

Write log equation.

Rewrite in exponent form.

Solve.

→ $\log_a(a^n) = n$



Oct 26-11:07 AM

Summary of properties of logarithms

Given that $a > 0$

- If $x = a^b$ then $\log_a x = b$
- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a b$ is undefined if b is negative
- $\log_a 0$ is undefined
- $\log_a (a^n) = n$



Oct 26-11:09 AM

Find the value of x if $\log_2 x = 5$

Answer

$$\log_2 x = 5$$

$$2^5 = x$$

$$x = 32$$

Rewrite in exponent form.

Solve.



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