4.8 Applications of exponential and logarithmic functions

Exponential growth and decay

Models of exponential growth and decay use exponential functions.

Here are just a few applications of exponential growth and decay models.

Biology

- Growth of micro-organisms in a culture
- Human population
- Spread of a virus

Physics

- Nuclear chain reactions
- Heat transfer

Economics

- Pyramid schemes

Computer technology

- Processing power of computers
- Internet traffic growth

You may wish to pick one of these as the basis of your Mathematical Exploration.

Nov 3-11:05 AM

Two areas of mathematics that appear to be completely disconnected might be exponentials and probability.

But consider this problem…

A group of people go to lunch and afterwards pick up their hats at random. What is the probability that no one gets their own hat?

It can be shown that this probability is $\frac{1}{e}$.

(You might like to explore this once you have studied probability further.)

Can you think of any other areas of knowledge that are surprisingly connected?

Nov 5-11:07 AM

The population, $A(t)$, in thousands, of a city is modeled by the function $A(t) = 30e^{(0.02)t}$ where t is the number of years after 2010. Use this model to answer these questions:

a What was the population of the city in 2010?

b By what percentage is the population of the city increasing each year?

c What will the population of the city be in 2020?

d When will the city's population be 60 000?

Answers

a $A(0) = 30e^0$
$= 30$
The population in 2010 was 30 000.

t is the number of years **after** *2010, so for 2010, t = 0*

b $A(1) = 30e^{(0.02)}$
$\frac{30e^{(0.02)}}{30} = e^{(0.02)}$
$= 1.0202...$
The population is increasing at 2.02% each year.

Write an equation for the population one year after 2010.
Calculate the multiplying factor.

c $A(10) = 30e^{(0.02)\times 10}$
$= 36.642...$
In 2020 the population will be 36 642

In 2020, t = 10

d $60 = 30e^{(0.02)t}$
$2 = e^{(0.02)t}$
$\ln 2 = \ln e^{(0.02)t}$
$\ln 2 = 0.02t$
$t = \frac{\ln 2}{0.02}$
$t = 34.657...$
The population will be 60 000 after 34.65 years, that is, during 2044.

When population is 60 000, A(t) = 60
Take logarithms of each side.
Bring down the exponent.
Solve for t.

Nov 5-11:08 AM

A casserole is removed from the oven and cools according to the model with equation $T(t) = 85e^{-0.1t}$, where t is the time in minutes and T is the temperature in °C.

a What is the temperature of the casserole when it is removed from the oven?

b If the temperature of the room is 25 °C, how long will it take for the casserole to reach room temperature?

Answers

a $T(0) = 85e^0$
$= 85$
The temperature of the casserole is 85 °C

When the casserole is removed from the oven, t = 0

b $85e^{-0.1t} = 25$
$e^{-0.1t} = \frac{25}{85} = \frac{5}{17}$
$\ln e^{-0.1t} = \ln \frac{5}{17}$
$= -1.22377...$
$t = 12.2$ (3 sf)
The casserole will reach room temperature after 12.2 min.

T = 25 if the temperature of the room is 25 °C.
Take logarithms of both sides.
Solve for t.

Nov 5-11:10 AM

WORK ON THE REVIEW!!

I will post some videos to help.

Nov 5-11:11 AM