CALCULUS

The Final Frontier

Feb 24-10:09 AM

I can define rate in mathematical terms.

Rates - examining rates of change plants the seeds of what we will call calculus.

What is a rate?

Rate = change in one variable with respect to another

Feb 24-10:13 AM

I can find examples of rates in term of their components.

Rates - examining rates of change plants the seeds of what we will call calculus.

What are some examples ?

velocity $= \frac{\Delta \text{ distance}}{\Delta \text{ time}}$

acceleration $= \frac{\Delta \text{ velocity}}{\Delta \text{ time}}$

slope (gradient) $= \frac{\Delta X}{\Delta Y}$

jerk $= \frac{\Delta \text{ acceleration}}{\Delta \text{ time}}$

Feb 24-10:13 AM

I can calculate rates of change.

Rates - examining rates of change plants the seeds of what we will call calculus.

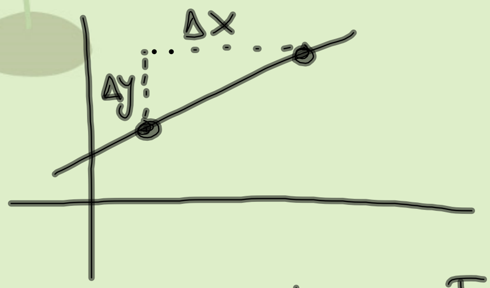
How do we calculate it ?

$$\frac{\underset{(y)}{\text{final value}} - \underset{(y)}{\text{initial value}}}{\underset{(x)}{\text{final time}} - \underset{(x)}{\text{initial time}}}$$

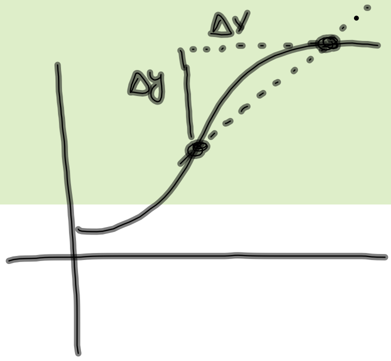
Feb 24-10:13 AM

I can calculate rates of change.

Linear Functions



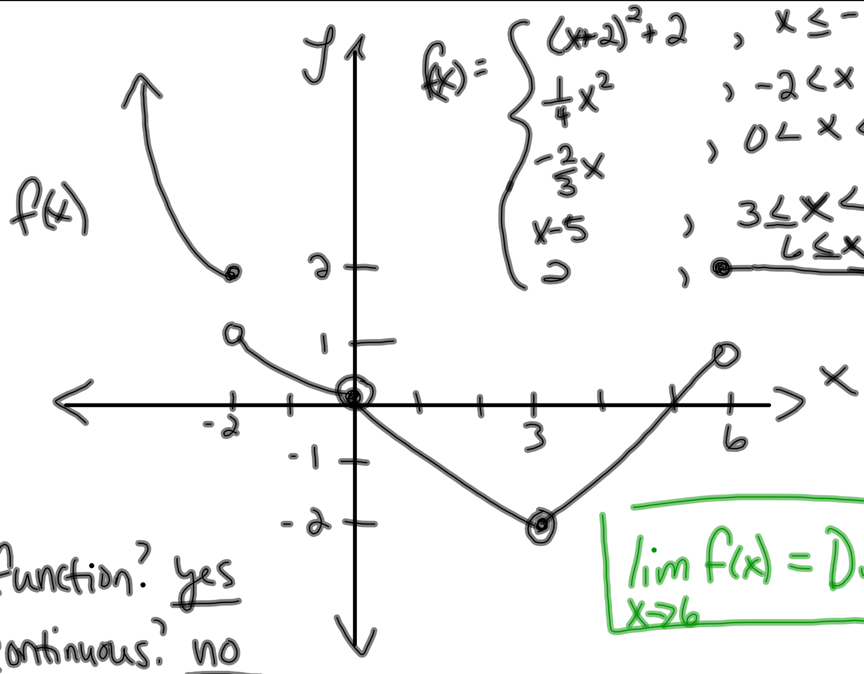
non-linear Functions



$\frac{\Delta y}{\Delta x}$ = avg rate of change

Feb 24-10:13 AM

$$f(x)=\begin{cases}(x+2)^2+2, & x\le -2\\ \frac{1}{4}x^2, & -2<x\le 0\\ -\frac{2}{3}x, & 0<x<3\\ x-5, & 3\le x<6\\ 2, & 6\le x\end{cases}$$



function? yes

continuous? no

Domain = $\mathbb{R}$

$\lim_{x\to 6} f(x) = DNE$

$\lim_{x\to 6^+} f(x) = 2$

not =

$\lim_{x\to 6^-} f(x) = 1$

Dec 21-1:04 PM

Definition 1 Let *f(x)* be a function defined on an interval that contains $x = a$, except possibly at $x = a$. Then we say that,

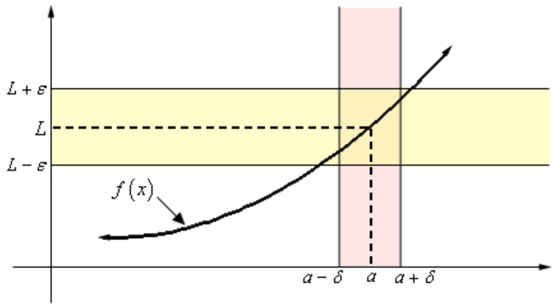
$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is some number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \qquad \text{whenever} \qquad 0 < |x - a| < \delta$$

Wow. That's a mouth full. Now that it's written down, just what does this mean?

Let's take a look at the following graph and let's also assume that the limit does exist.



What the definition is telling us is that for **any** number $\varepsilon > 0$ that we pick we can go to our graph and sketch two horizontal lines at $L+\varepsilon$ and $L-\varepsilon$ a shown on the graph above. Then somewhere out there in the world is another number $\delta > 0$, which we will need to determine, that will allow us to add in two vertical lines to our graph at $a+\delta$ and $a-\delta$.

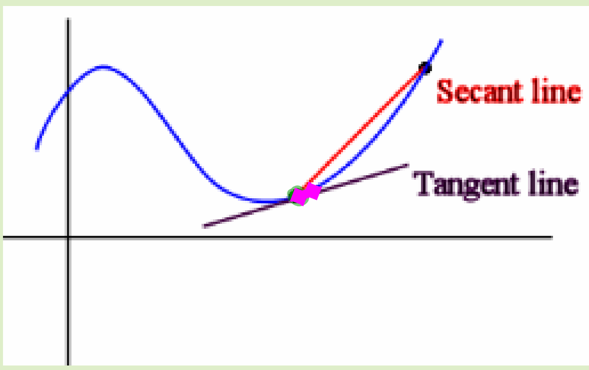
Now, if we take any x in the pink region, *i.e.* between $a+\delta$ and $a-\delta$, then this x will be closer to a than either of $a+\delta$ and $a-\delta$. Or,

$$|x - a| < \delta$$

Dec 14-10:07 AM

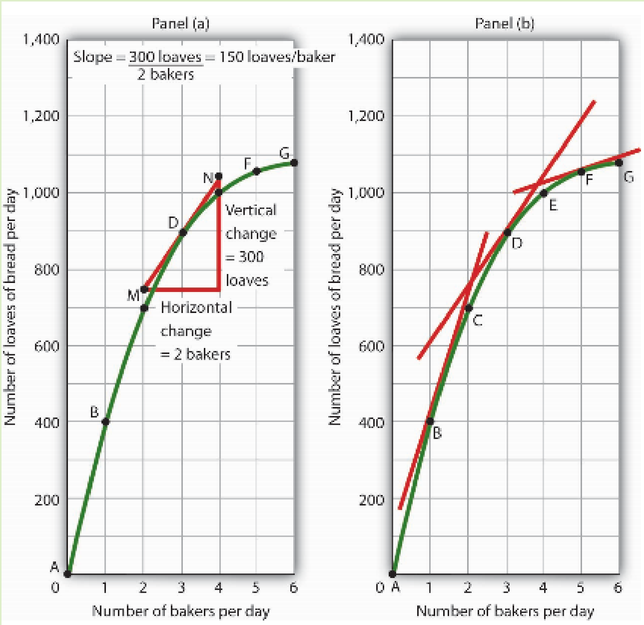
Non-Linear Functions

To estimate the rate of change for a non-linear function we will create a linear approximation of the function at the local level.



Feb 24-10:13 AM

Non-Linear Functions



I can estimate rates of change.

Feb 24-10:13 AM