OPTIMIZATION

① $f(x)=\dfrac{2x}{x^2+4}$

a) $f'(x)=\dfrac{(x^2+4)(2)-(2x)(2x)}{(x^2+4)^2}$

$f'(x)=\dfrac{2x^2+8-4x^2}{(x^2+4)^2}$

$f'(x)=\dfrac{-2x^2+8}{(x^2+4)^2}$

$f'(x)=\dfrac{-2(x^2-4)}{(x^2+4)^2}$

$0=\dfrac{-2(x^2-4)}{(x^2+4)^2}$

$0=-2(x^2-4)$

$0=x^2-4$

$0=(x+2)(x-2)$

$f'(1)=$ $x=-2$ $x=2$

x	1	2	3
$f'(x)$	$6/25$	0	$-10/169$
shape	↗	→	↘

$\therefore$ max

x	-3	-2	1
$f'(x)$	$-10/169$	0	$6/25$
shape	↘	→	↗

$\therefore$ min at $x=-2$

$f(-2)=\dfrac{2(-2)}{(-2)^2+4}$

$=\dfrac{-4}{8}$

$=-\dfrac{1}{2}$

Jan 12-12:01 PM

OPTIMIZATION

② $f(x)=xe^{-x}$

a) $f'(x)=x(e^{-x})(-1)+(e^{-x})(1)$

$=-xe^{-x}+e^{-x}$

$=e^{-x}(-x+1)$

b) $0=e^{-x}(-x+1)$

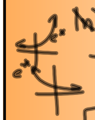
$e^{-x}=0$ | $-x+1=0$

$1=x$

$\cancel{\ln e^{-x}}=\ln 0$

$-x=$ undef

no soln



c) $\boxed{f''(1)=?}$

$f'(x)=e^{-x}(-x+1)$

$f''(x)=e^{-x}(-1)+(-x+1)(e^{-x})(-1)$

$=-e^{-x}+xe^{-x}-e^{-x}$

$=-2e^{-x}+xe^{-x}$

$f''(x)=e^{-x}(-2+x)$

$f''(1)=e^{-1}(-2+1)$

$=e^{-1}(-1)$

$\therefore$ max $=-\dfrac{1}{e}$

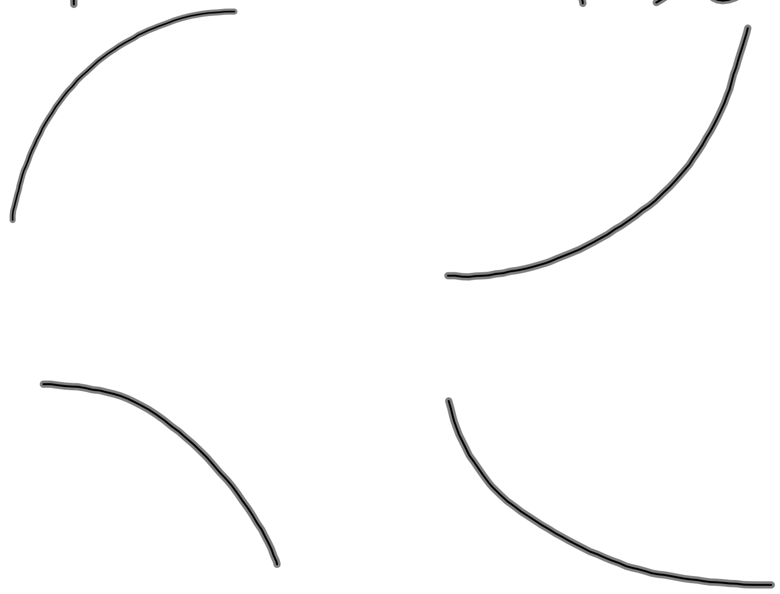
Jan 12-12:01 PM

Concave down $f'' < 0$

Concave up $f'' > 0$

increasing $f' > 0$

decreasing $f' < 0$



Jan 21-1:04 PM