CALCULUS

The Final Frontier

Feb 24-10:09 AM

I can define rate in mathematical terms.

Rates - examining rates of change plants the seeds of what we will call calculus.

What is a rate?

Rate = change in one variable with respect to another

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I can find examples of rates in term of their components.

Rates - examining rates of change plants the seeds of what we will call calculus.

What are some examples ?

$\text{velocity} = \dfrac{\Delta \text{dist}}{\Delta \text{time}}$ $\text{text rate} = \dfrac{\#\text{ texts}}{\text{minute}}$

$\text{production rate} \quad \dfrac{\#\text{ of items produced}}{\text{time period (day)}}$ growth rate

pay rate

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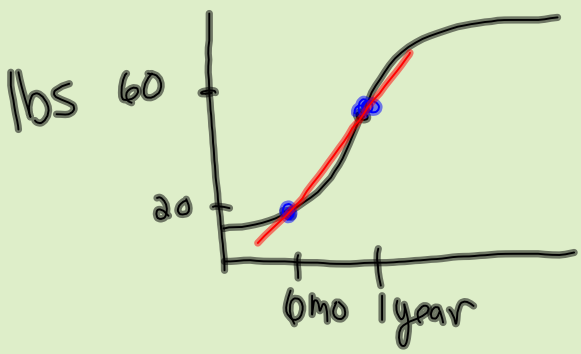
I can calculate rates of change.

Rates - examining rates of change plants the seeds of what we will call calculus.

How do we calculate it ?



Growth rate from 6 mo to 1 year

$$\frac{\Delta \text{Growth}}{\Delta \text{time}} = \frac{60-20}{12-6} = \frac{40}{6} = 6.67 \ \frac{\text{lb}}{\text{mo}}$$

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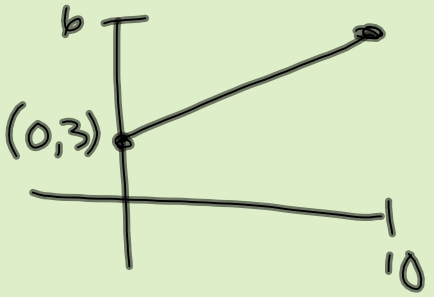
I can calculate rates of change.

Linear Functions

$y = 2x + 3$ $m = 2$



$m = \frac{3}{10} = .3$

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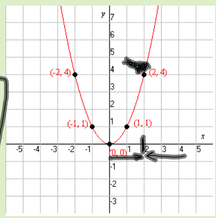
Use a GDC to examine each function graphically and numerically.
Find the limit or state that it does not exist.

a $\lim_{x \to 2} x^2$ **b** $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ **c** $\lim_{x \to 0} f(x)$; where $f(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases}$

a.)

$\boxed{\lim_{x \to 2} x^2 = 4}$



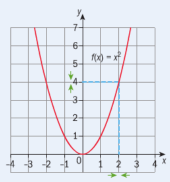
$\lim_{x \to 2^-} x^2 = 4$

$\lim_{x \to 2^+} x^2 = 4$

Answers

a $\lim_{x \to 2} x^2$

Plot the graph of $f(x) = x^2$ using a GDC, and look at the values of $f(x)$ as x approaches 2 from the right and from the left.



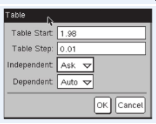
Graphically, $f(x)$ approaches 4 as x approaches 2:
Numerically, as x becomes close to 2 from either side, $f(x)$ becomes close to 4.

$\rightarrow 2 \leftarrow$

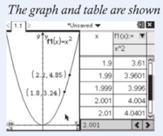
x	1.8	1.9	1.99	1.999	2.001	2.01	2.1	2.2
f1(x)	3.24	3.61	3.960	3.996	4.004	4.040	4.41	4.84

$\rightarrow 4 \leftarrow$

To build the table above using a GDC, enter $f1(x) = x^2$. Then set the independent variables to 'Ask'. Enter the values for x.



The graph and table are shown on the same screen.



For $f(x) = x^2$ we can substitute and find that $\lim_{x \to 2} x^2 = 2^2 = 4$

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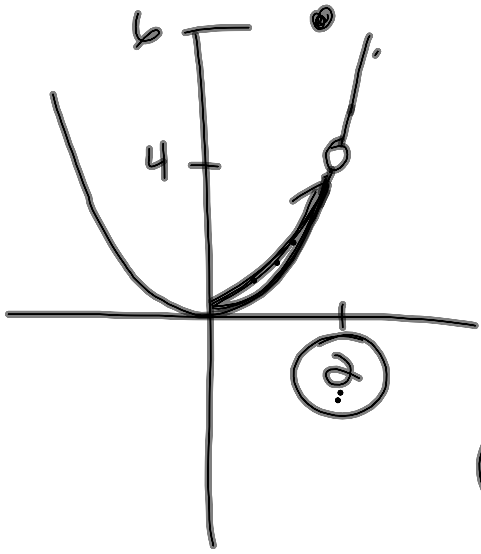

(2,6)

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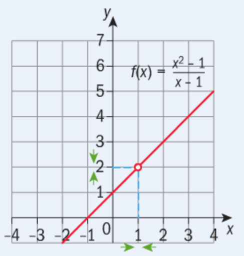
Use a GDC to examine each function graphically and numerically.
Find the limit or state that it does not exist.

a $\lim_{x\to2} x^2$ **b** $\lim_{x\to1}\frac{x^2-1}{x-1}$ **c** $\lim_{x\to0} f(x)$; where $f(x)=\begin{cases}1 & \text{for } x\geq0\\ -1 & \text{for } x<0\end{cases}$

b.)

$$\lim_{x\to1}\frac{x^2-1}{x-1}=2$$



$$\frac{x^2-1}{x-1}=\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

hole at $x=1$

Since division by zero is not defined, $f(x)=\frac{x^2-1}{x-1}$ *is undefined when* $x-1=0$ *or* $x=1$*. Therefore there is a* **discontinuity** *in the graph when* $x=1$*. Notice that* $f(x)=\frac{x^2-1}{x-1}=\frac{(x+1)(x-1)}{x-1}=x+1$*, when* $x\neq1$

Even though $f(x)=\frac{x^2-1}{x-1}$ *is undefined when* $x=1$*, the limit exists since as* x *becomes close to 1 from either side,* $f(x)$ *becomes close to 2.*

$\rightarrow 1 \leftarrow$

x	0.8	0.9	0.99	0.999	1.001	1.01	1.1	1.2
f(x)	1.8	1.9	1.99	1.999	2.001	2.01	2.1	2.2

$\rightarrow 2 \leftarrow$

Note that $\lim_{x\to1}\frac{x^2-1}{x-1}=\lim_{x\to1}\frac{(x+1)(x-1)}{x-1}$

$$=\lim_{x\to1}(x+1)=1+1=2$$

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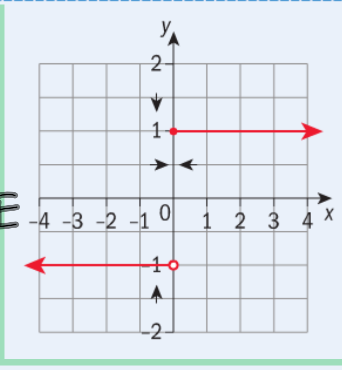
Use a GDC to examine each function graphically and numerically.
Find the limit or state that it does not exist.

a $\lim_{x\to 2} x^2$ **b** $\lim_{x\to 1}\frac{x^2-1}{x-1}$ **c** $\lim_{x\to 0} f(x)$; where $f(x)=\begin{cases}1 & \text{for } x\geq 0\\ -1 & \text{for } x<0\end{cases}$

c.)

$\lim_{x\to 0} f(x) = DNE$



$\lim_{x\to 0^+} f(x) = 1$

$\lim_{x\to 0^-} f(x) = -1$

f(x) does not approach the same value as x approaches 0 from the left and right:

$\rightarrow 0 \leftarrow$

x	−0.2	−0.1	−0.01	−0.001	0.001	0.01	0.1	0.2
f(x)	−1	−1	−1	−1	1	1	1	1

Note that f(0) = 1, but $\lim_{x\to 0} f(x)$ *does not exist.*

This is because f (x) is close to 1 for values of x to the right of x = 0 and f(x) is close to −1 for values of x to the left of x = 0.

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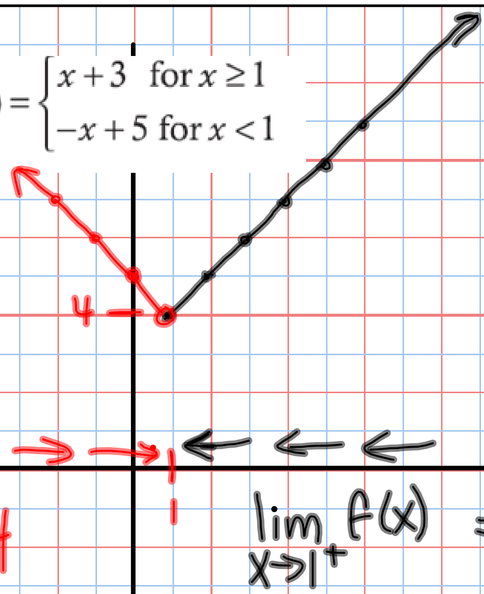
5 $\lim_{x\to 1} f(x)$; where $f(x)=\begin{cases}x+3 & \text{for } x\geq 1\\ -x+5 & \text{for } x<1\end{cases}$



$\lim_{x\to 1^-} f(x) = 4$

$\lim_{x\to 1^+} f(x) = 4$

$\therefore \lim_{x\to 1} f(x) = 4$

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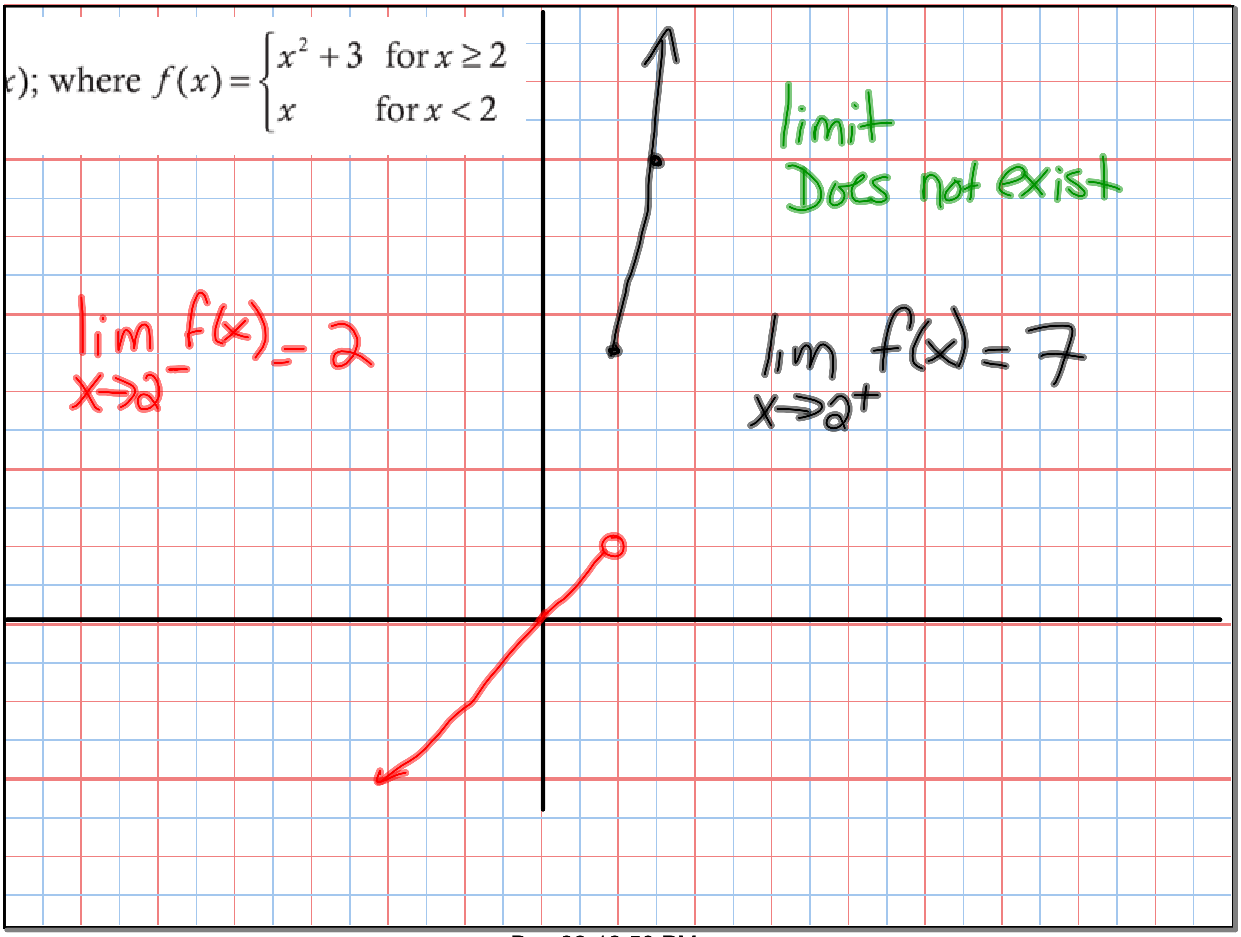
$x)$; where $f(x)=\begin{cases} x^2+3 & \text{for } x \geq 2 \\ x & \text{for } x<2 \end{cases}$



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Exercise 7B

Use a GDC to examine each function graphically and numerically. Find the limit or state that it does not exist.

1 $\lim_{x\to 3}(x^2+1)$

2 $\lim_{x\to 0}\dfrac{x^3-4x^2+x}{x}$

3 $\lim_{x\to 2}\dfrac{x^2-3x+2}{x-2}$

4 $\lim_{x\to 4}\dfrac{1}{x-4}$

5 $\lim_{x\to 1} f(x)$; where $f(x)=\begin{cases} x+3 & \text{for } x \geq 1 \\ -x+5 & \text{for } x<1 \end{cases}$

6 $\lim_{x\to 2} f(x)$; where $f(x)=\begin{cases} x^2+3 & \text{for } x \geq 2 \\ x & \text{for } x<2 \end{cases}$

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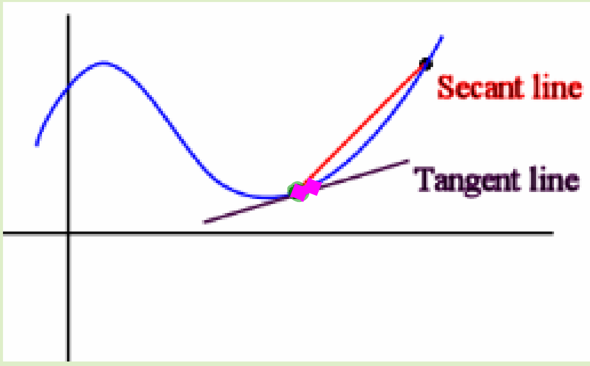
Non-Linear Functions

To estimate the rate of change for a non-linear function we will create a linear approximation of the function at the local level.



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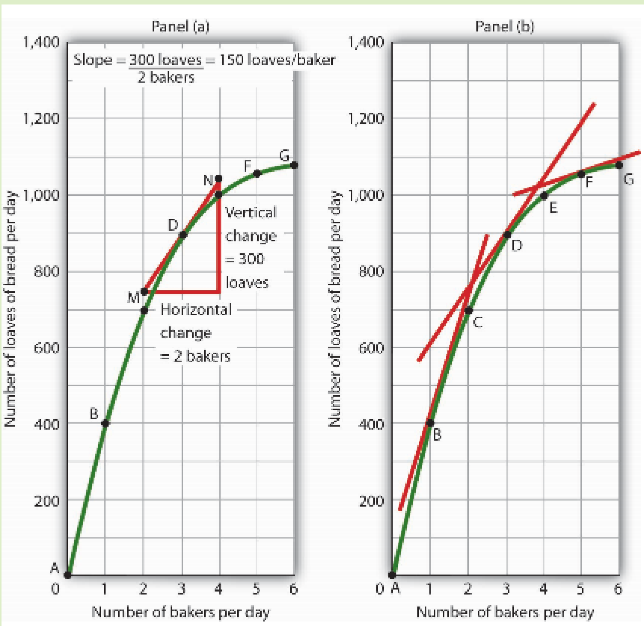
Non-Linear Functions



I can estimate rates of change.

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