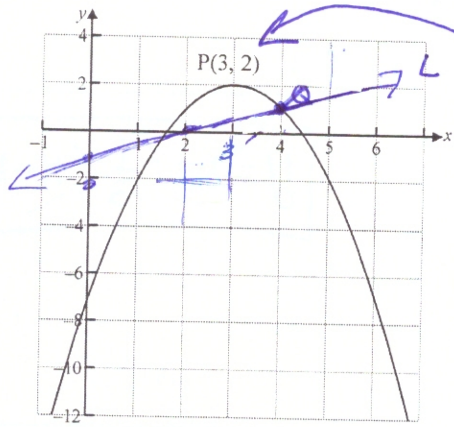
**IB Math: Calculus Test** January 17th, 2016

**Name:** Key

1. The function $f(x)$ is defined as $f(x) = -(x-h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P(3, 2).




(a) Write down the value of

(i) $h$; $\boxed{3}$

(ii) $k$. $\boxed{2}$

} vertex form uses the vertex point

**(2)**

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

$f(x) = -(x-3)^2 + 2$

$f(x) = -(x^2 - 6x + 9) + 2$

$f(x) = -x^2 + 6x - 9 + 2$

✓ $f(x) = -x^2 + 6x - 7$

**(1)**

(c) Find $f'(x)$. $\boxed{f'(x) = -2x + 6}$

**(2)**

The point Q lies on the curve and has coordinates (4, 1). A straight line $L$, through Q, is perpendicular to the tangent at Q.

(d) (i) Calculate the gradient of $L$.

$f'(4) = -2(4) + 6$
$= -8 + 6$
$= -2$ $\rightarrow \perp$ slope $= \boxed{\frac{1}{2}}$

(ii) Find the equation of $L$.

point $= (4, 1)$
slope $= \frac{1}{2}$

$\boxed{y - 1 = \frac{1}{2}(x-4)}$

(iii) The line $L$ intersects the curve again at R. Find the $x$-coordinate of R.

Algebraically

$y - 1 = \frac{1}{2}(x-4)$ set =

or $y = -x^2 + 6x - 7$

use graphing calculator (above)

$\boxed{(1.5, -.25)}$

$-x^2 + 6x - 7 = \frac{1}{2}(x-4) + 1$

$-x^2 + 6x - 7 = \frac{1}{2}x - 2 + 1$

$0 = x^2 - 5.5x + 6$

$x = \frac{5.5 \pm \sqrt{5.5^2 - 4(1)(6)}}{2}$

$\rightarrow x = 1.5, 4$

$f(1.5) = -.25$

**(8)**

**(Total 13 marks)**

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2. Find the equation of the <u>normal</u> to the curve with equation

$$y = x^3 + 1$$ → perpendicular

at the point (1,2).

*Working:*

$$\frac{dy}{dx} = 3x^2$$

$$\boxed{y - 2 = -\frac{1}{3}(x - 1)}$$

when $x = 1$ $\quad m = 3(1)^2$

$m = 3$

$\perp$ slope $= -\frac{1}{3}$

*Answers:*

$y - 2 = -\frac{1}{3}(x - 1)$

**(Total 4 marks)**

3. The function $g(x)$ is defined for $-3 \le x \le 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

| $x$ | $-3 < x < -2$ | $-2$ | $-2 < x < 1$ | $1$ | $1 < x < 3$ |
|---|---|---|---|---|---|
| $g'(x)$ | negative | 0 | positive | 0 | negative |
| | ↘ | → | ↗ | → | ↘ |

Basic shape:

| $x$ | $-3 < x < -\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2} < x < 3$ |
|---|---|---|---|
| $g''(x)$ | positive | 0 | negative |
| | $\cup$ | | $\cap$ |

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of $x$ for which $g$ has a maximum.

max at $x = 1$

**(2)**

(b) On which intervals is the value of $g$ decreasing?

$g$ is decreasing: $-3 < x < -2$ and $1 < x < 3$

**(2)**

(c) Write down the value of $x$ for which the graph of $g$ has a point of inflexion.

inflection point when $x = -\frac{1}{2}$

**(2)**

**(Total 6 marks)**

**4.** Differentiate with respect to $x$:

(a) $(x^2 + 1)^2$

(b) $e^{sinx}$

*Working:*

a) $y = (x^2+1)^2$

$y' = 2(x^2+1)^1(2x)$

$\boxed{y' = 4x(x^2+1)}$

b) $y = e^{\sin x}$

$y' = (e^{\sin x})(\cos x)$

or $\boxed{y' = (\cos x)(e^{\sin x})}$

*Answers*:

(a) $4x(x^2+1)$ ..............................

(b) $(\cos x)(e^{\sin x})$ ..............................

**(Total 4 marks)**

**6.**

Let $g(x) = \dfrac{\ln x}{x^2}$, for $x > 0$.

$$g'(x) = \frac{(x^2)(\frac{1}{x}) - (\ln x)(2x)}{(x^2)^2}$$

$$g'(x) = \frac{x - 2x\ln x}{x^4}$$

$$\boxed{g'(x) = \frac{1 - 2\ln x}{x^3}}$$

(a) Use the quotient rule to show that $g'(x) = \dfrac{1 - 2\ln x}{x^3}$.

**(4)**

(b) The graph of $g$ has a maximum point at A. Find the $x$-coordinate of A.

**(3)**

**(Total 7 marks)**

max pt is where $g' = 0$

$$0 = \frac{1 - 2\ln x}{x^3}$$

$0 = 1 - 2\ln x$

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$e^{\frac{1}{2}} = x$

now get y

$$g(e^{\frac{1}{2}}) = \frac{\ln(e^{\frac{1}{2}})}{(e^{\frac{1}{2}})^2}$$

$$y = \frac{\frac{1}{2}}{e}$$

$\frac{1}{2} \cdot \frac{1}{e} = \frac{1}{2e}$

$$\boxed{\left(e^{\frac{1}{2}}, \frac{1}{2e}\right)}$$