

1. Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

* // to $y = 5x$ means $m = 5x$ so derivative = 5 (Total 4 marks)

$$\begin{aligned}y &= x^2 - x \\y' &= 2x - 1 \\5 &= 2x - 1 \\6 &= 2x \\3 &= x\end{aligned}$$

substitute

$$\begin{aligned}y &= x^2 - x \\y &= (3)^2 - 3 \\y &= 9 - 3 \\y &= 6\end{aligned}$$

$$(3, 6)$$

2. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

* Max point means $y' = 0$ and $y'' > 0$

$$\begin{aligned}y &= x^3 - 10x^2 + 12x + 23 \\y' &= 3x^2 - 20x + 12 \\0 &= 3x^2 - 20x + 12 \\0 &= (3x-2)(x-6)\end{aligned}$$

$$\begin{array}{l|l}0 = 3x - 2 & 0 = x - 6 \\2 = 3x & 6 = x \\\frac{2}{3} = x &\end{array}$$

only this one is between -1 and 3

$$y'' = 6x - 20$$

$$\text{at } x = \frac{2}{3} \quad y'' = 6\left(\frac{2}{3}\right) - 20$$

$$y'' = 4 - 20$$

$$y'' = -16$$

\therefore maximum b/c concave down

3. A curve has equation $y = x(x-4)^2$.

(a) For this curve find

(i) the x -intercepts;

$$x=0 \quad | \quad x=4 \quad | \quad x=4$$

(ii) the coordinates of the maximum point;

(iii) the coordinates of the point of inflection.

$$\begin{array}{l|l}y &= x(x-4)^2 \\y' &= x(2(x-4)(1)+(x-4)^2(1)) \\y' &= 2x(x-4) + (x-4)^2 \\y' &= (x-4)(2x+x-4) \\y' &= (x-4)(3x-4)\end{array}$$

1st der

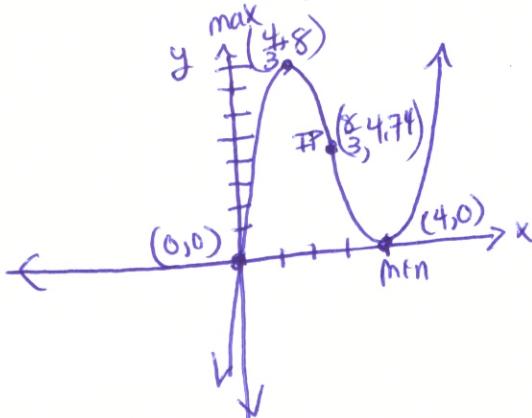
$$y'' = (x-4)(3) + (3x-4)(1)$$

$$\begin{aligned}y &= \left(\frac{2}{3}\right)^3 - 10\left(\frac{2}{3}\right)^2 + 12\left(\frac{2}{3}\right) + 23 \\y &= \frac{725}{27} \\&\text{max pt} = \left(\frac{2}{3}, \frac{725}{27}\right)\end{aligned}$$

(9)

(b) Use your answers to part (a) to sketch a graph of the curve for $0 \leq x \leq 4$, clearly indicating the features you have found in part (a).

(b)



$$\begin{array}{l|l}y'' &= 3x-12+3x-4 \\y'' &= 6x-16\end{array}$$

2nd der.

$$\begin{array}{l|l}0 = (x-4)(3x-4) & 16 = 6x \\X=4 & X = \frac{16}{6} \\Y=0 & Y \approx 9.48 \\Y'' = 8 & Y'' = -8 \\& \therefore \curvearrowleft \\& \therefore \curvearrowright \\& \min @ (4,0) \quad \max @ (\frac{4}{3}, 8)\end{array}$$

(iii) $0 = 6x - 16$ (3)

$$16 = 6x$$

(Total 12 marks)

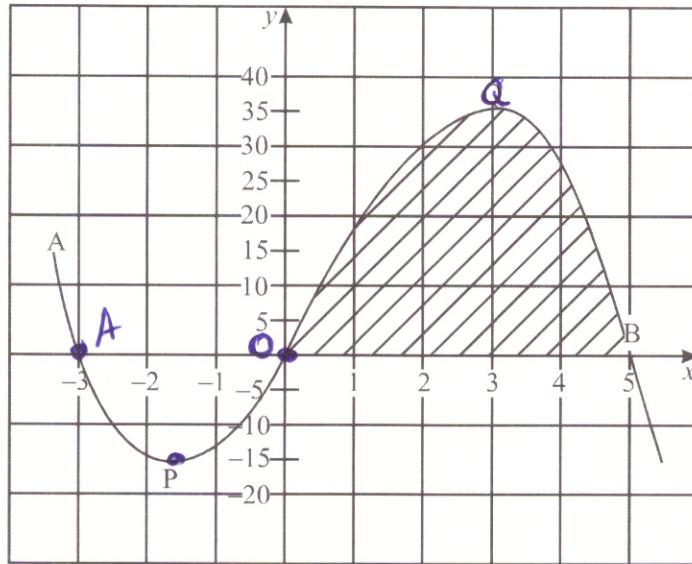
$$\frac{8}{3} = x$$

at $x = \frac{8}{3}$ $y \approx 4.74$

inflection pt at $(\frac{8}{3}, 4.74)$ (B of)

4. The diagram below shows part of the graph of the function

$$f: x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x -axis at $A(-3, 0)$, $B(5, 0)$ and the origin, O . There is a minimum point at P and a maximum point at Q .

- (a) The function may also be written in the form $f: x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a ; $a = -3$ $f(x) = -x(x+3)(x-5)$

(ii) b ; $b = 5$ $f(x) = -x(x^2 - 2x - 15)$

- (b) Find

(i) $f'(x)$; $f'(x) = 3x^2 + 4x + 15$

(ii) the exact values of x at which $f'(x) = 0$; $0 = -3x^2 - 4x - 15$

(iii) the value of the function at Q . $f(3) = 3(3+3)(3-5) = 3(6)(-2) = 36$ $\begin{array}{|c|c|} \hline x & = -\frac{5}{3} & x = 3 \\ \hline \text{min} & & \text{max} \\ \hline \end{array}$

- (c) (i) Find the equation of the tangent to the graph of f at O .

- (ii) This tangent cuts the graph of f at another point. Give the x -coordinate of this point.

→ (i) $f'(x) = -3x^2 + 4x + 15$
 $f'(0) = 15$

$m = 15$

point $= (0, 0)$

$y - 0 = 15(x - 0)$

$y = 15x$

→ (ii) $15x = -x^3 + 2x^2 + 15x$
 $(\text{set them } = \text{ and solve})$

$$\begin{aligned} 0 &= -x^3 + 2x^2 \\ 0 &= x^2(-x+2) \\ x=0 &\quad | -x+2=0 \\ &\quad | 2=x \end{aligned}$$

$f(2) = 30$

$\boxed{(2, 30)}$

(2)

(7)

(4)

(Total 13 marks)

5. A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2, t \geq 0$$

- (a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

initial height means time = 0 so $h = 2 + 20(0) - 5(0)^2$ (2)

- (b) Show that the height of the ball after one second is 17 metres.

*at $t=1$ $h = 2 + 20(1) - 5(1)^2$
 $h = 2 + 20 - 5$
 $h = 17$* (2)

- (c) At a later time the ball is **again** at a height of 17 metres.

- (i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.

$$17 = 2 + 20t - 5t^2$$

- (ii) Solve the equation **algebraically**.

$$17 = 2 + 20t - 5t^2 \quad (4)$$

$$\begin{aligned} 5t^2 - 20t + 15 &= 0 \\ t^2 - 4t + 3 &= 0 \\ (t-3)(t-1) &= 0 \end{aligned}$$

$t=3$ $t=0$

- (d) (i) Find $\frac{dh}{dt}$. $h = 2 + 20t - 5t^2$

$$\frac{dh}{dt} = 20 - 10t$$

- (ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

$$V = \frac{dh}{dt} \text{ at } t=0 \quad \frac{dh}{dt} = 20 - 10(0)$$

$$\frac{dh}{dt} = 20$$

- (iii) Find **when** the ball reaches its maximum height.

** this is notation for 2nd derivative max height = when $\frac{dh}{dt} = 0$ and $\frac{d^2h}{dt^2} < 0$*

$$0 = 20 - 10t$$

$$10t = 20$$

$$t = 2$$

$\frac{d^2h}{dt^2} = -10$
 \therefore graph is
 always concave
 down so when
 $t=2$ it is a max value
 (7)

- (iv) Find the maximum height of the ball.

$$\text{When } t=2 \quad h = 2 + 20(2) - 5(2)^2$$

$$h = 2 + 40 - 20$$

$$h = 22$$

(Total 15 marks)

6. Differentiate with respect to x :

$$(a) \ln(3x-1) \rightarrow \frac{1}{3x-1} (3) = \boxed{\frac{3}{3x-1}}$$

$$(b) \sqrt{3-4x} = (3-4x)^{\frac{1}{2}} \rightarrow \frac{1}{2}(3-4x)^{-\frac{1}{2}} (-4) \\ = \boxed{-2(3-4x)^{-\frac{1}{2}}} \text{ or } \boxed{\frac{-2}{\sqrt{3-4x}}}$$

$$(c) (2x+5)^3 \rightarrow 3(2x+5)^2 (2) = \boxed{6(2x+5)^2}$$

(Total 6 marks)

7. A particle moves along a straight line. When it is a distance s from a fixed point, where $s > 1$, the velocity v is given by $v = \frac{(3s+2)}{(2s-1)}$. Find the acceleration when $s = 2$.

$$\text{acceleration} = v' \quad \text{so ...}$$

(Total 4 marks)

$$v' = \frac{(2s-1)(3) - (3s+2)(2)}{(2s-1)^2}$$

$$v' = \frac{6s-3 - (6s+4)}{(2s-1)^2}$$

$$v' = \frac{6s-3-6s-4}{(2s-1)^2}$$

$$v' = \frac{-7}{(2s-1)^2}$$

when $s=2$

$$v' = \frac{-7}{(2(2)-1)^2}$$

$$v' = \frac{-7}{3^2}$$

$$v' = \frac{-7}{9}$$

$$\boxed{a = \frac{-7}{9}}$$